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The Hubble tension in the GW era: A Cross-Correlation study

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Outline

- Hubble tension
- Gravitational Waves opportunities
- Cross-Correlation study
- Outlook

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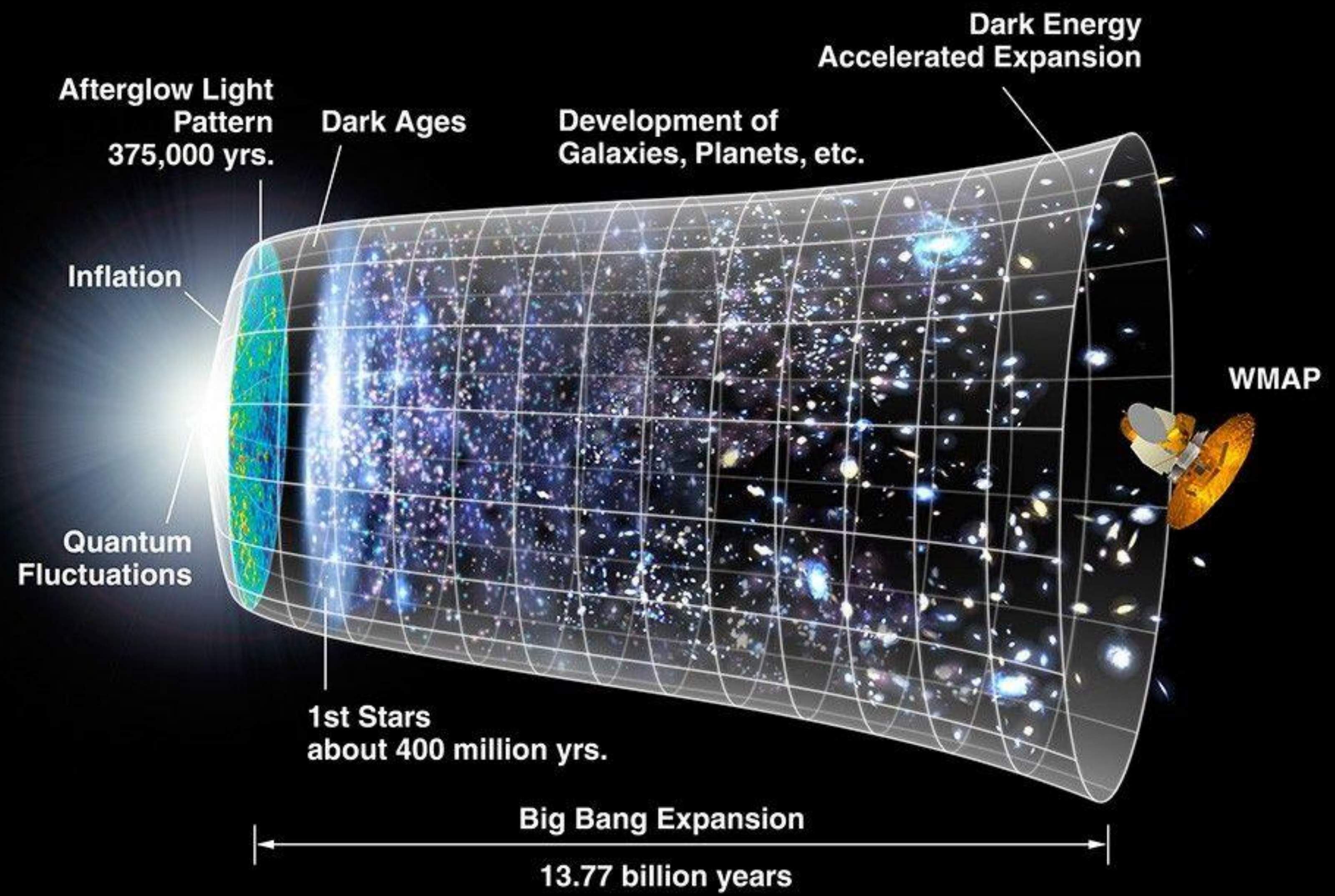
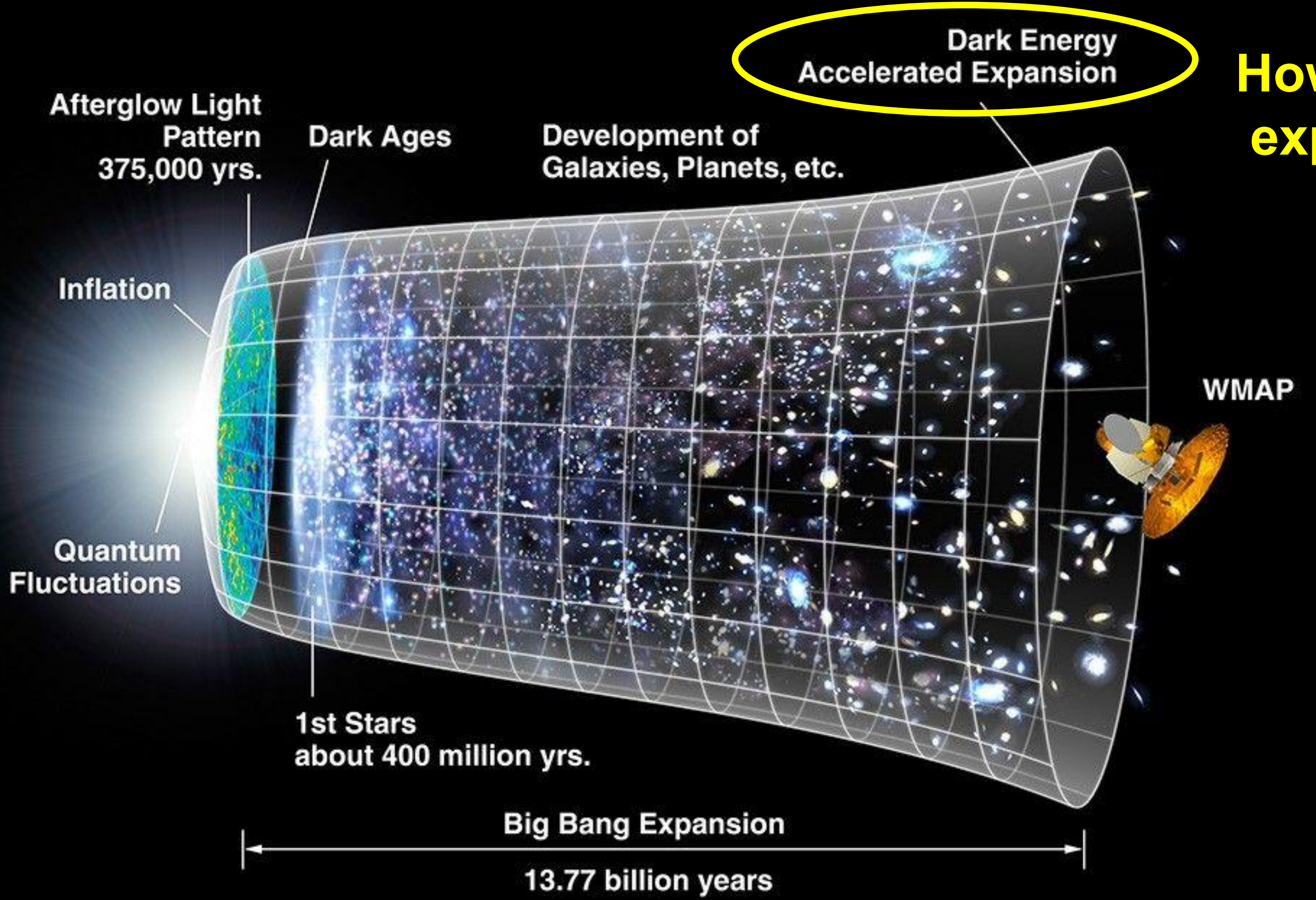
- **Hubble tension**
- Gravitational Waves opportunities
- Cross-Correlation study
- Outlook

Universe evolution



NASA/WAMP Science team

Universe evolution



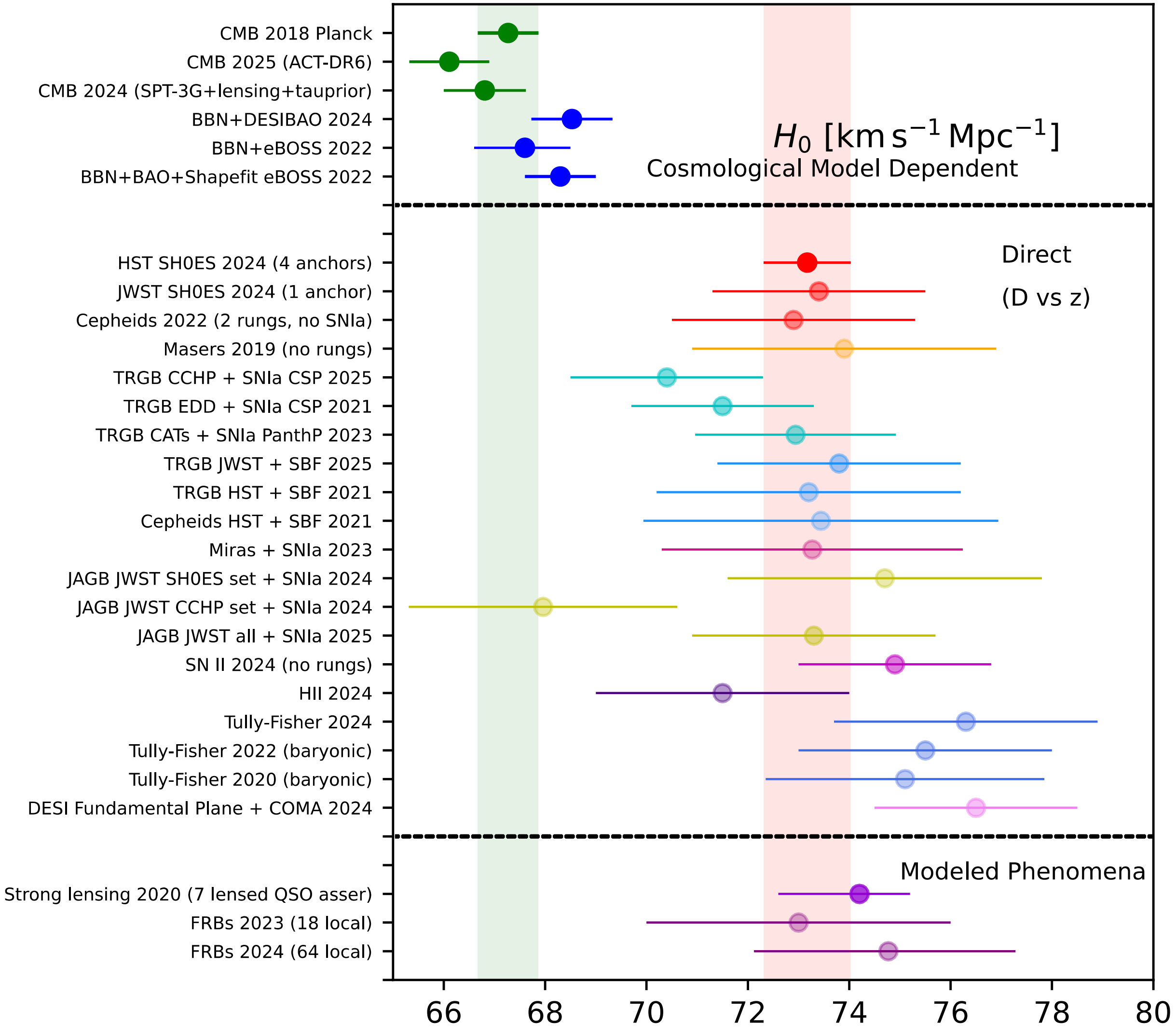
How fast is it expanding?

$$H = \frac{\dot{a}}{a}$$

NASA/WAMP Science team

Hubble parameter

- Early Universe measurement using **CMB**;
  - ➢ Cosmological model dependent;
- Late Universe measurements:
  1. With **standard candles**;
     - ➢ Requires distance ladder;
  2. With **modeled phenomena**;
     - ➢ Astrophysical model dependent.



Di Valentino et al. 2504.01669

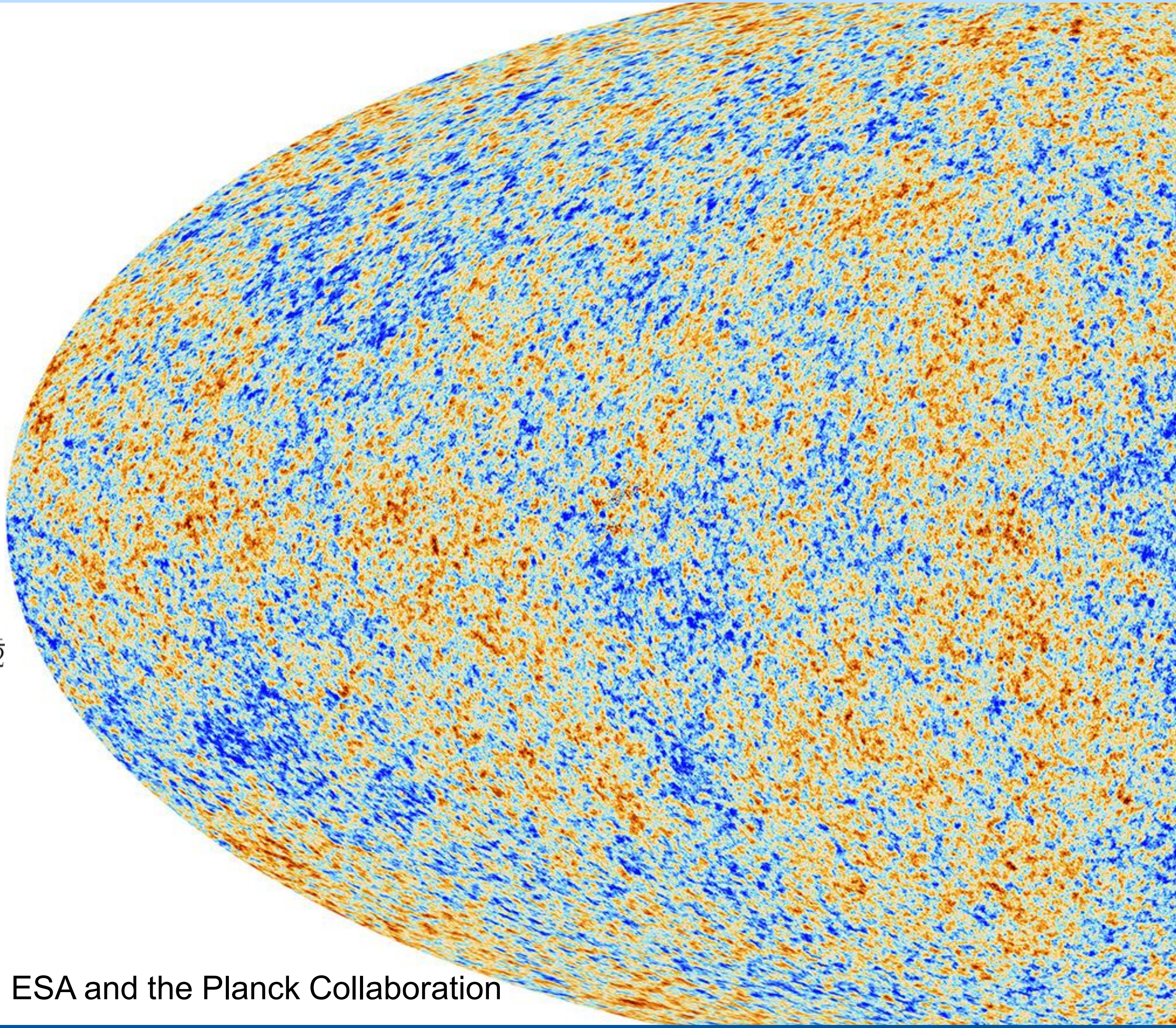
H_0 from CMB

Standard ruler at z=1100. CMB provide a measurement of H_0 based on the angular size of perturbations:

$$D_A(z) = \frac{1}{1+z}\int_0^z \frac{dz'}{H(z')}$$

$$H(z) = H_0\left[\Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_k(1+z)^2 + \Omega_\Lambda\right]^{\frac{1}{2}}$$

Dependencies on GR, flatness, ΛCDM, cosmological evolution.



ESA and the Planck Collaboration

H_0 from standard candles

$$D_L = a(t_0)(1+z)\int_0^z \frac{c\,dz'}{a(t_0)H(z')}$$

➢ **Redshift z**

- Spectroscopy (e.g. H lines);
- Photometry (e.g. galaxies color);
- 21cm;
- *Supernova Type Ia,* regular light curve.
- …

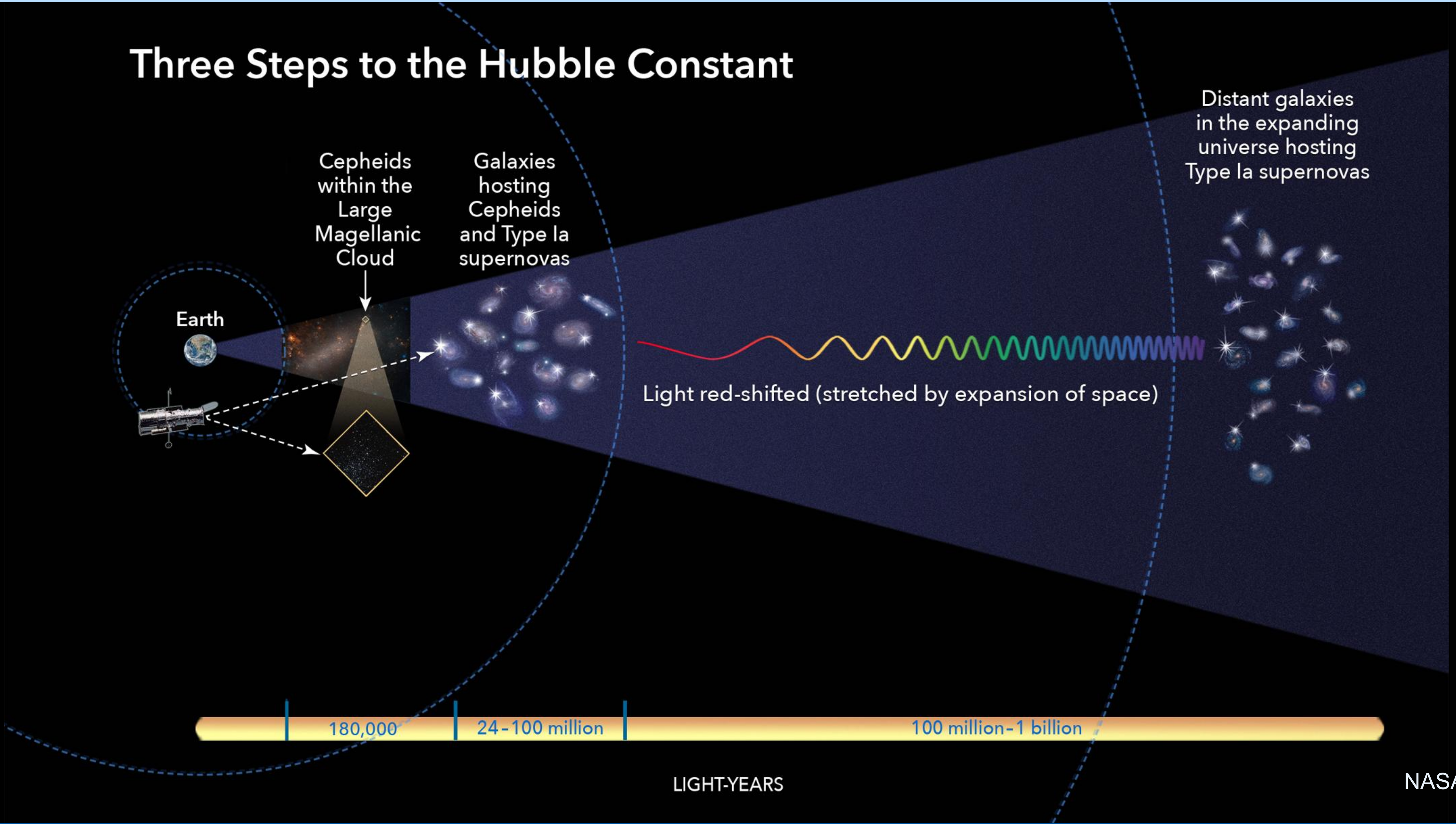
➢ **Luminosity distance D_L**

Using **cosmic distance ladder**:

Cepheids: regularly pulsating star, used to calibrate the luminosity of

Supernovae Type Ia: predictable peak luminosity and bright.

Distance ladder



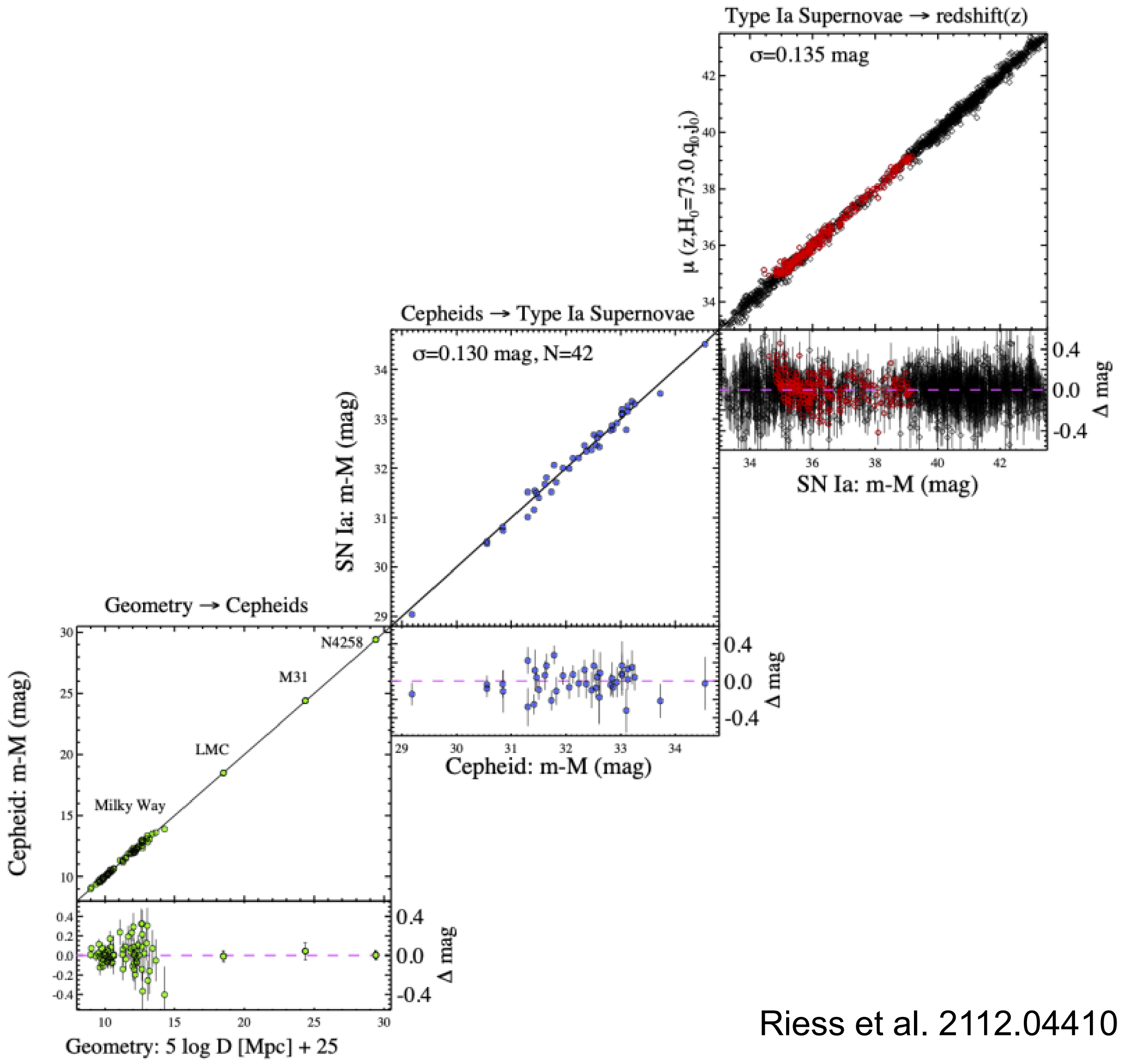
NASA, ESA, and A. Feild (STScI)

H_0 from standard candles

$$D_L = a(t_0)(1+z)\int_0^z \frac{c\,dz'}{a(t_0)H(z')}$$

- **Redshift z**
- **Luminosity distance D_L**

- Pro: Low redshifts, hence model independent measurement;
- Cons: It needs lot of calibration and systematics.



Riess et al. 2112.04410

Hubble tension

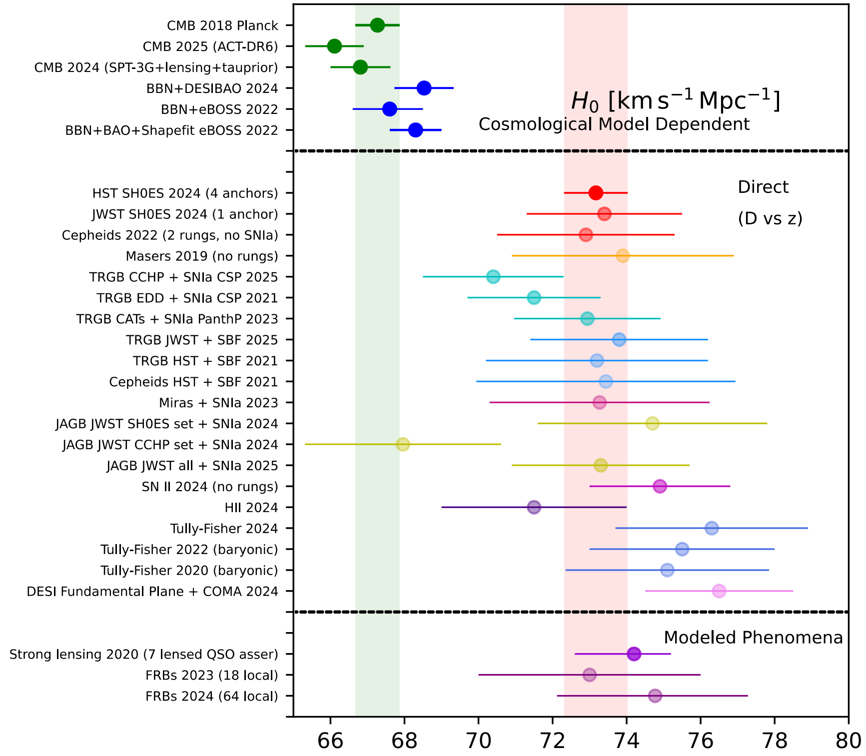
$5 - 6\sigma$ tension

- Early Universe (Planck, ACT, SPT):

$$67.4 \pm 0.5 \text{ km s}^{-1}$$

- Late Universe (SH0ES):

$$73.17 \pm 0.86 \text{ km s}^{-1}$$



Di Valentino et al. 2504.01669

How can we solve this tension?

- New cosmological models
- New techniques to measure H_0

Can we measure H_0 with GWs?

Can we measure H_0 with GWs?

YES

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- **Gravitational Waves opportunities**
- Cross-Correlation study
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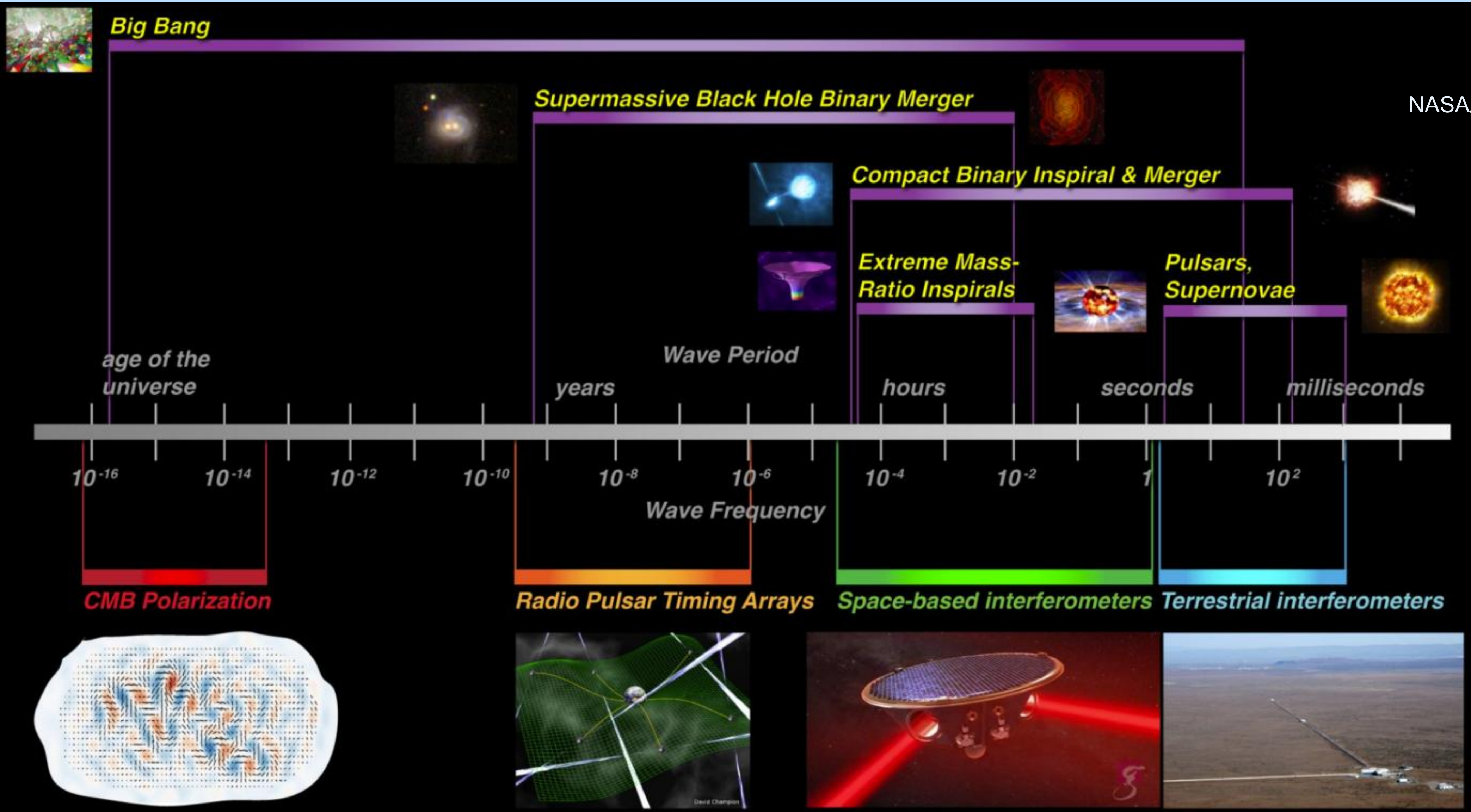
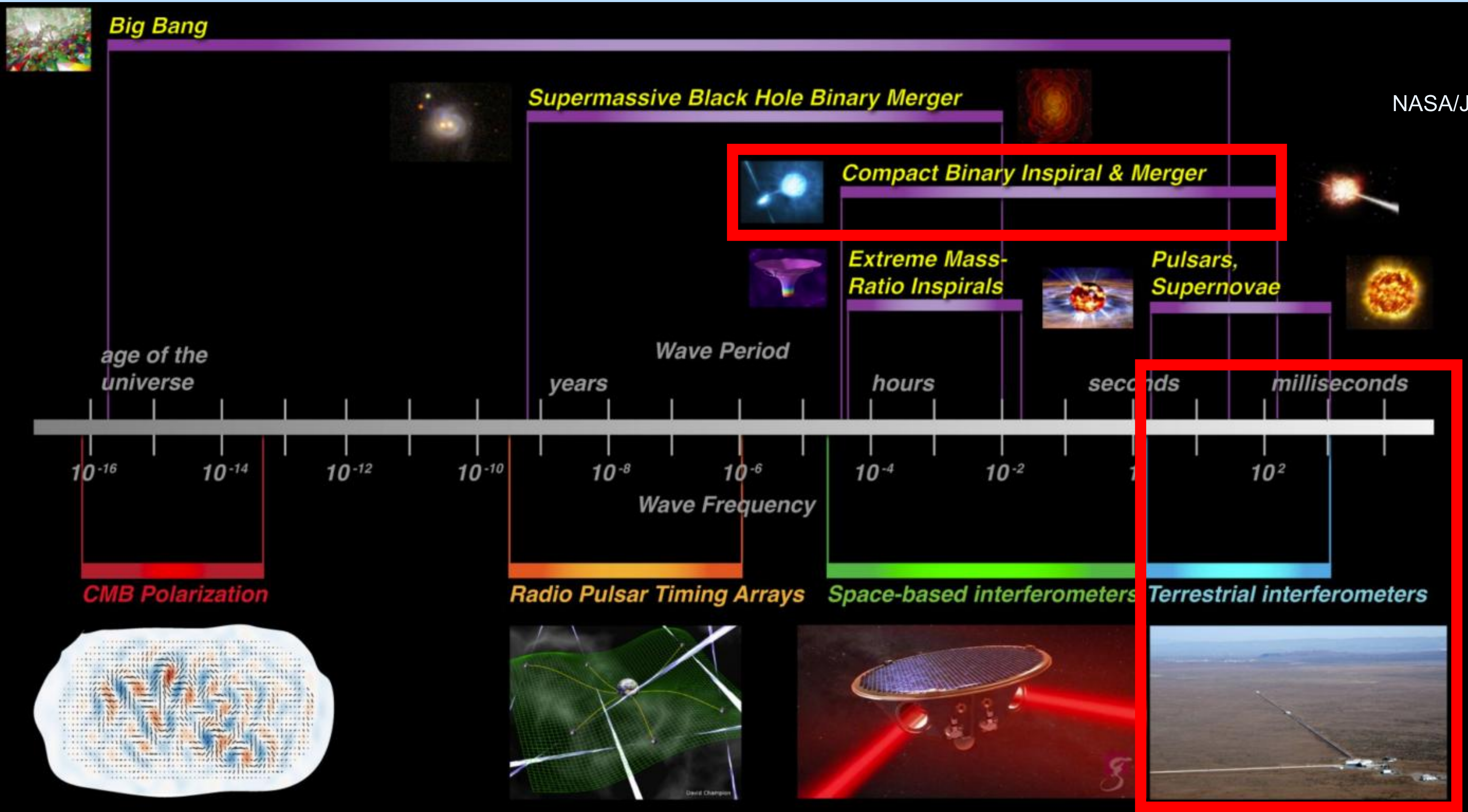
GWs spectrum



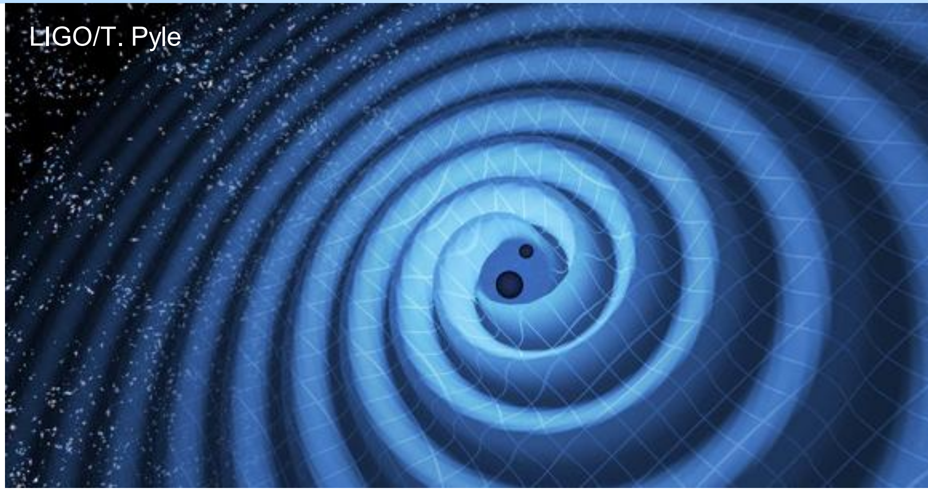
NASA/J. I.Thorpe

GWs spectrum

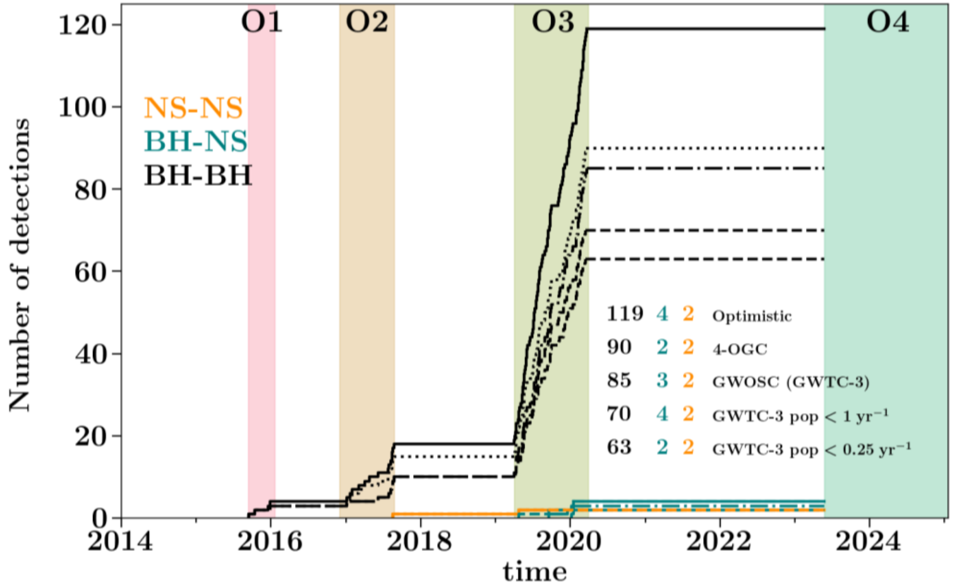
NASA/J. I.Thorpe



GW detections



- LIGO-Virgo-Kagra (LVK) started detecting GWs a decade ago;
- Observed GW signal comes from Compact Binaries Coalescence (CBC);
- Most measured events are called *dark sirens* due to the lack of complementary observations;
- The number of detections is rapidly growing.



Broekgaarden et al. 2303.17628

Standard sirens

$$D_L = a(t_0)(1+z)\int_0^z \frac{c\,dz'}{a(t_0)H(z')}$$

- **Luminosity distance D_L**

Compact objects binary are **standard sirens.**

$$h_+(t) = \frac{2\mathcal{M}_c^{5/3}\pi^{2/3} f_{GW}(t)^{2/3}}{D_L}\left[1+\cos\iota^2\right]\cos\Phi(t)$$

The strain of GW gives information on the luminosity distance.

$$h_\times(t) = \frac{4\mathcal{M}_c^{5/3}\pi^{2/3} f_{GW}(t)^{2/3}}{D_L}\cos\iota\sin\Phi(t)$$

Standard sirens

$$D_L = a(t_0)(1+z)\int_0^z \frac{c\,dz'}{a(t_0)H(z')}$$

- **Luminosity distance D_L**

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Disclaimer: luminosity distance is degenerate with the inclination of a binaries ι. This degeneracy can be broken at higher order.

H_0 from GWs

$$D_L = a(t_0)(1+z)\int_0^z \frac{c\,dz'}{a(t_0)H(z')}$$

➢ **Luminosity distance D_L**

Direct and independent measurement of luminosity distance.

➢ **Redshift z**

?

Redshift from GWs

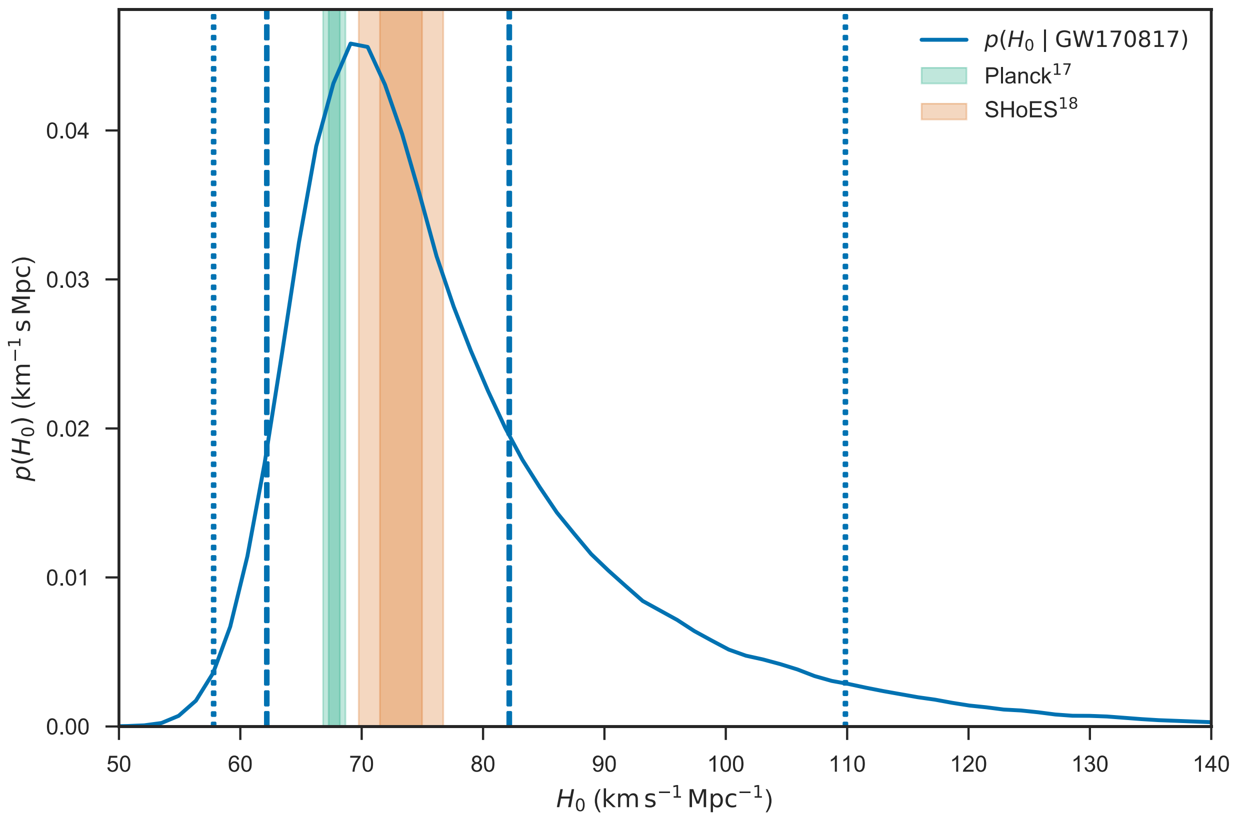
$$D_L = a(t_0)(1+z)\int_0^z \frac{c\,dz'}{a(t_0)H(z')}$$

- **Bright sirens**

  Multi-messenger observation of a coalescence (e.g. GW and EM).

  Redshift is inferred from the complementary measurement to GW.

  It suffers a lack of detections, it only happened once so far GW170817.



Jin et al. 2507.12965

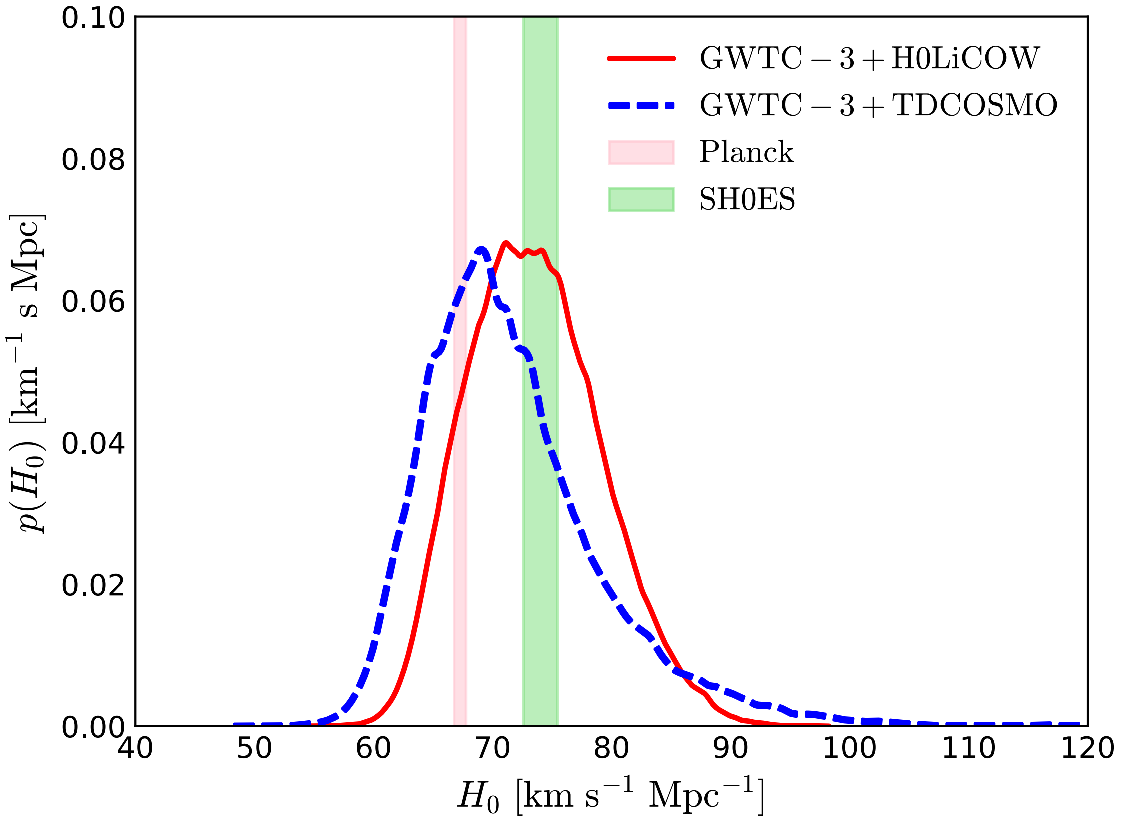
Redshift from GWs

$$D_L = a(t_0)(1+z)\int_0^z \frac{c\,dz'}{a(t_0)H(z')}$$

- Bright sirens
- **Dark sirens and galaxy association**

GW events are associated to a potential host galaxy, or set of galaxies, from which the redshift is inferred.

This method suffers from significant systematics.



Jin et al. 2507.12965

Redshift from GWs

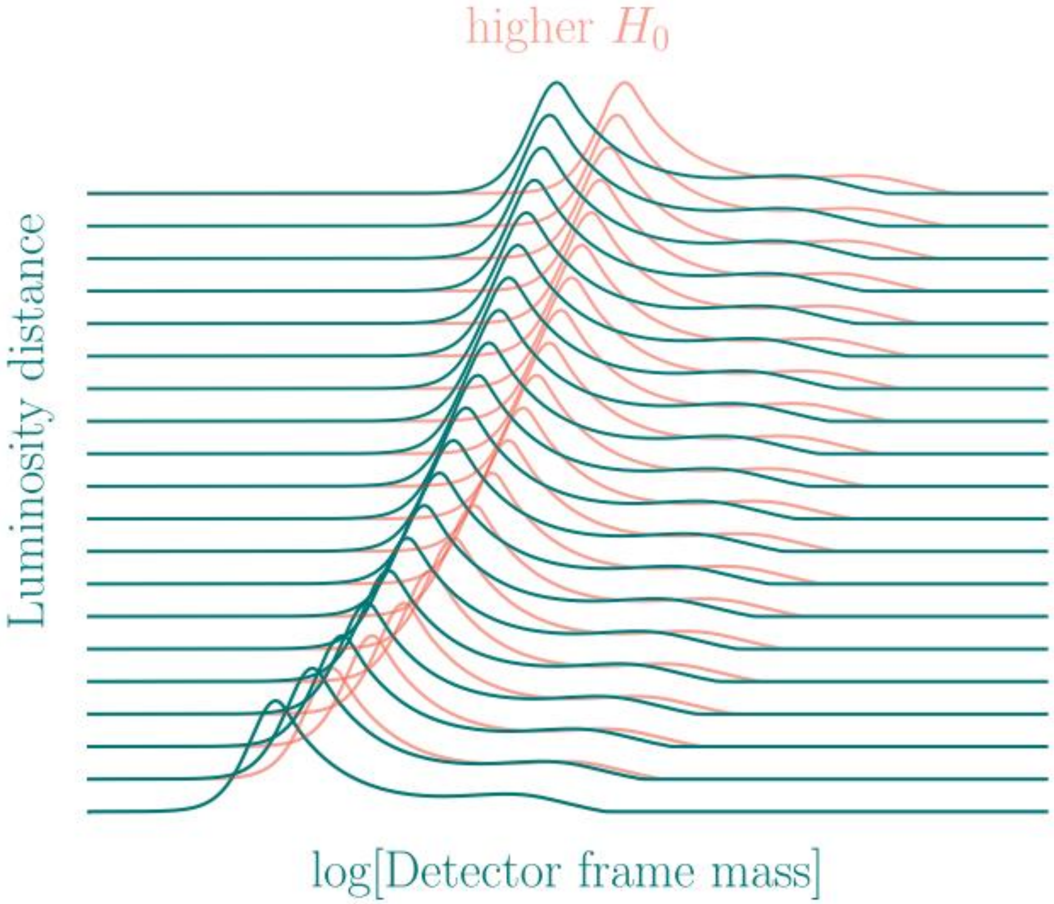
$$D_L = a(t_0)(1+z)\int_0^z \frac{c\,dz'}{a(t_0)H(z')}$$

- Bright sirens
- Dark sirens and galaxy association
- **Spectral sirens**

Distance redshifts the observed mass function of compact binaries.

$$\mathcal{M}_c^{\rm detector} = (1+z)\mathcal{M}_c^{\rm source}$$

The mass function needs to be known.



Pierra et al. 2507.10597

Redshift from GWs

$$D_L = a(t_0)(1+z)\int_0^z \frac{c\,dz'}{a(t_0)H(z')}$$

- Bright sirens
- Dark sirens and galaxy association
- Spectral sirens
- **Love sirens**

GW waveform analysis can be exploit for studying the tidal deformation of neutron stars close to merger, which can provide information on the redshift.

These effect are difficult to detect and tend to be degenerate with PN 3 and PN 3.5 phase terms.

Redshift from GWs

$$D_L = a(t_0)(1+z)\int_0^z \frac{c\,dz'}{a(t_0)H(z')}$$

- Bright sirens
- Dark sirens and galaxy association
- Spectral sirens
- Love sirens
- **Cross-Correlation with galaxies**

Statistical method to infer cosmological properties. With the only assumption that both galaxies and GW events are tracer of matter overdensities, we get both distance measurements from LSS.

Outline

- Hubble tension
- Gravitational Waves opportunities
- **Cross-Correlation study**
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Inferring cosmological parameters from galaxy and dark sirens cross-correlation

Giona Sala, Alessandro Cuoco, Julien Lesgourgues, Konstantinos-Rafail Revis, Lorenzo Valbusa Dall'Armi and Santiago Casas,
arXiv:2510.08699 [astro-ph.CO].

Cross-correlation statistically infers cosmological parameter by correlating two observables of the same object, in this case matter overdensities.

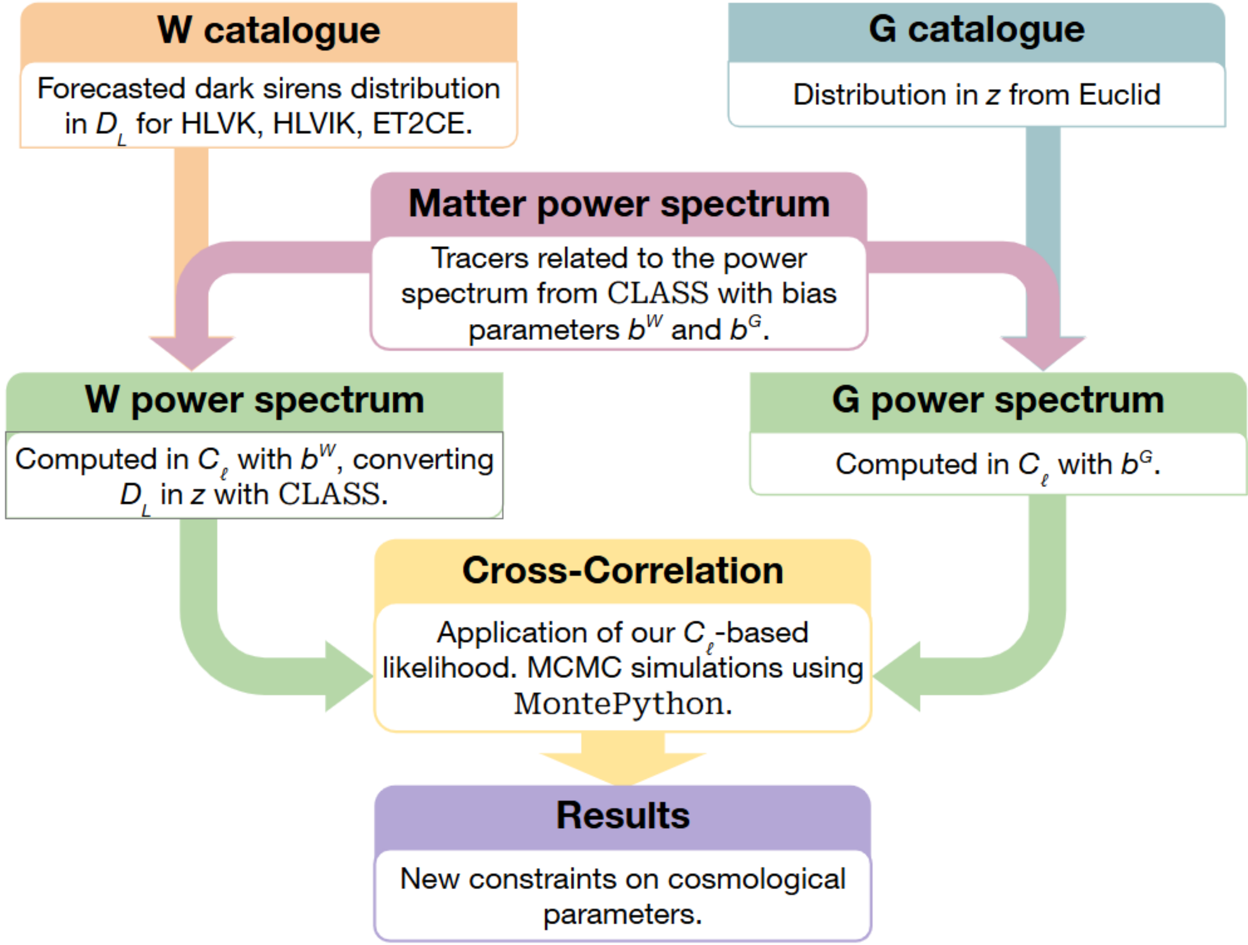
Pros: it only requires one assumption, that both galaxies and GWs trace LSS;

Cons: it requires a large sample of GW observations.

Roadmap

Procedure:

- Estimate GW measurements;
- Compute angular power spectrum C_ℓ;
- Cross-correlation for C_ℓ-based likelihood;
- Run MCMC chains to infer cosmological parameters.



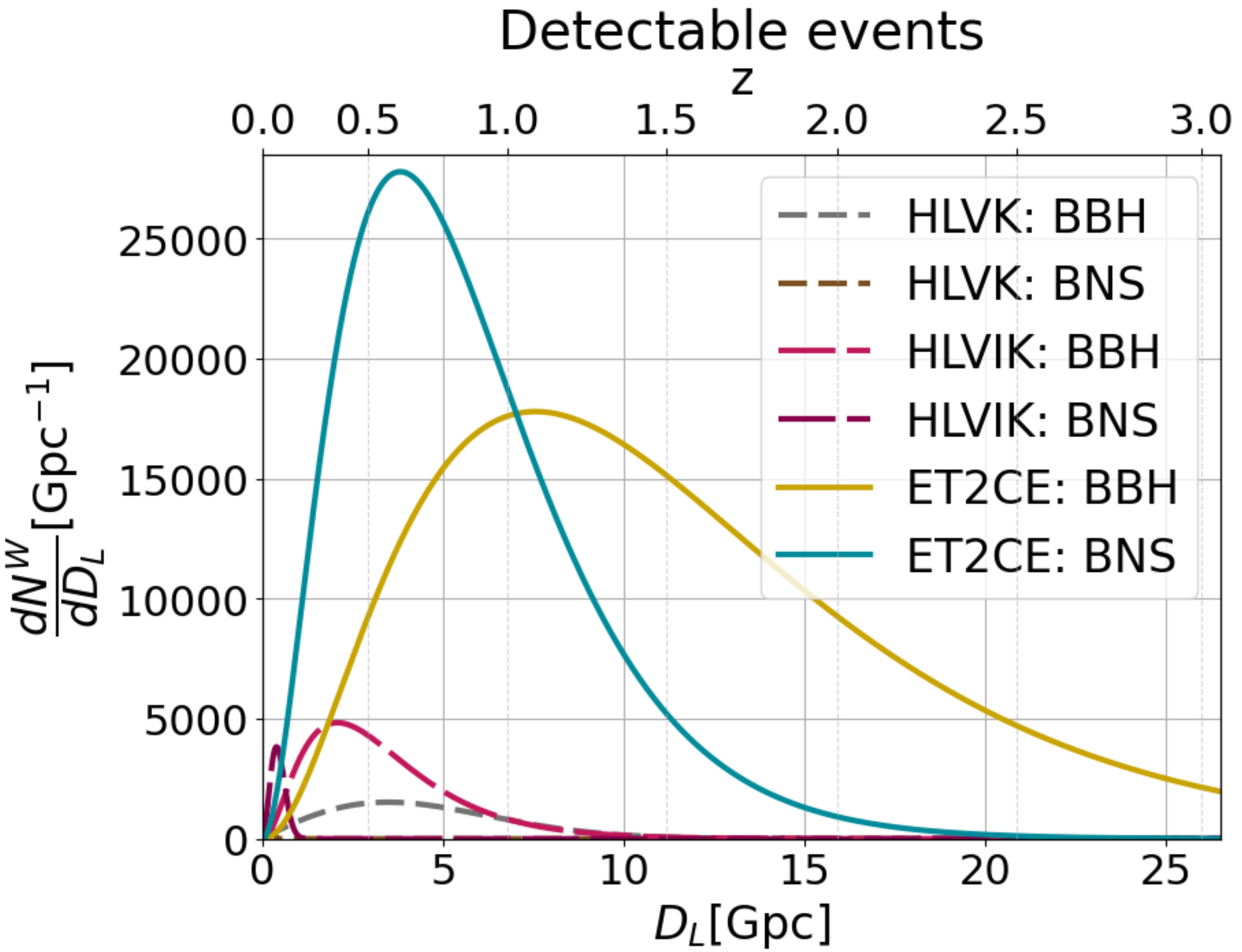
Mock data

Galaxy clustering:

- Euclid photometric forecasted data;

Dark sirens detections over 10 years:

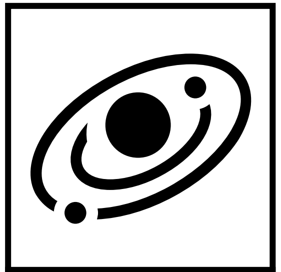
- HLVK: Advanced LVK;
- HLVIK: HLVK + LIGO India;
- ET2CE: Einstein Telescope + 2 Cosmic Explorer.



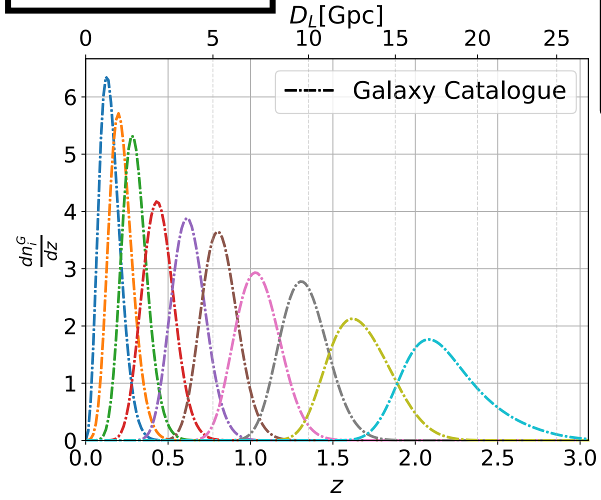
Tomographic approach

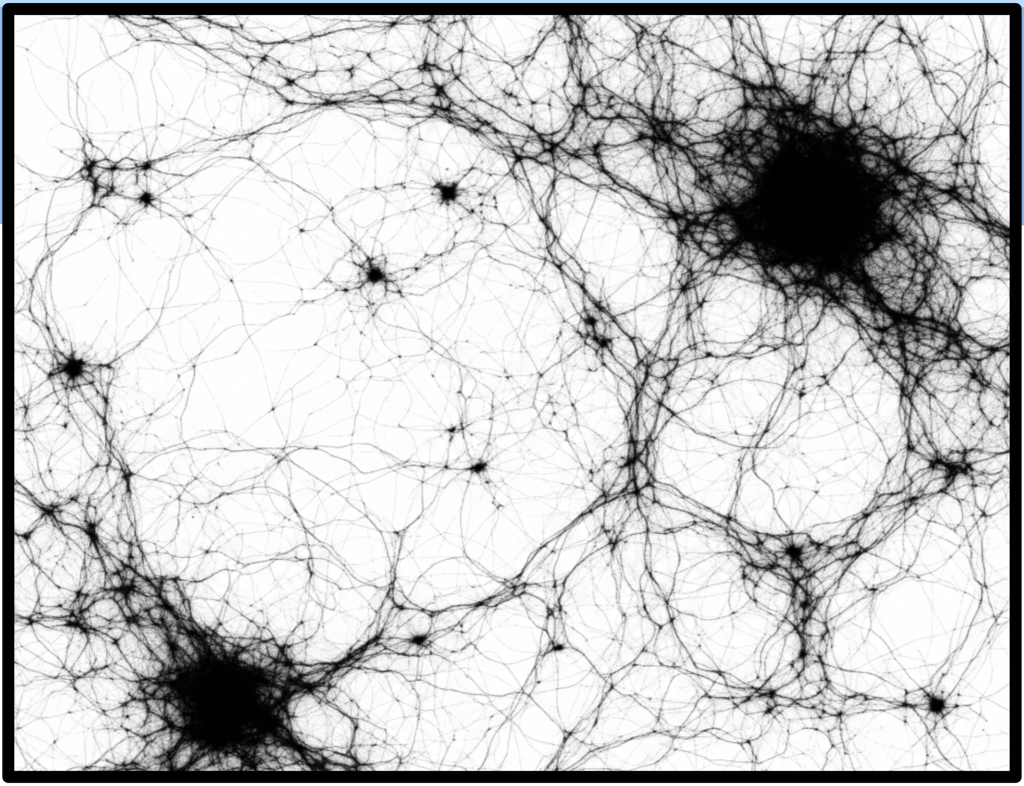
Redshift

z



galaxy clustering





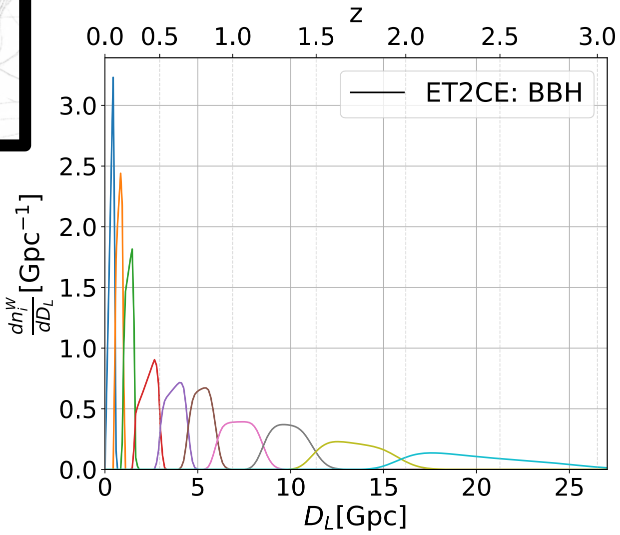
Binned and normalized distribution with error on the distance

$$\frac{dn^{G_i}}{dz}, \frac{dn^{W_i}}{dD_L}$$

Luminosity distance

D_L

dark sirens



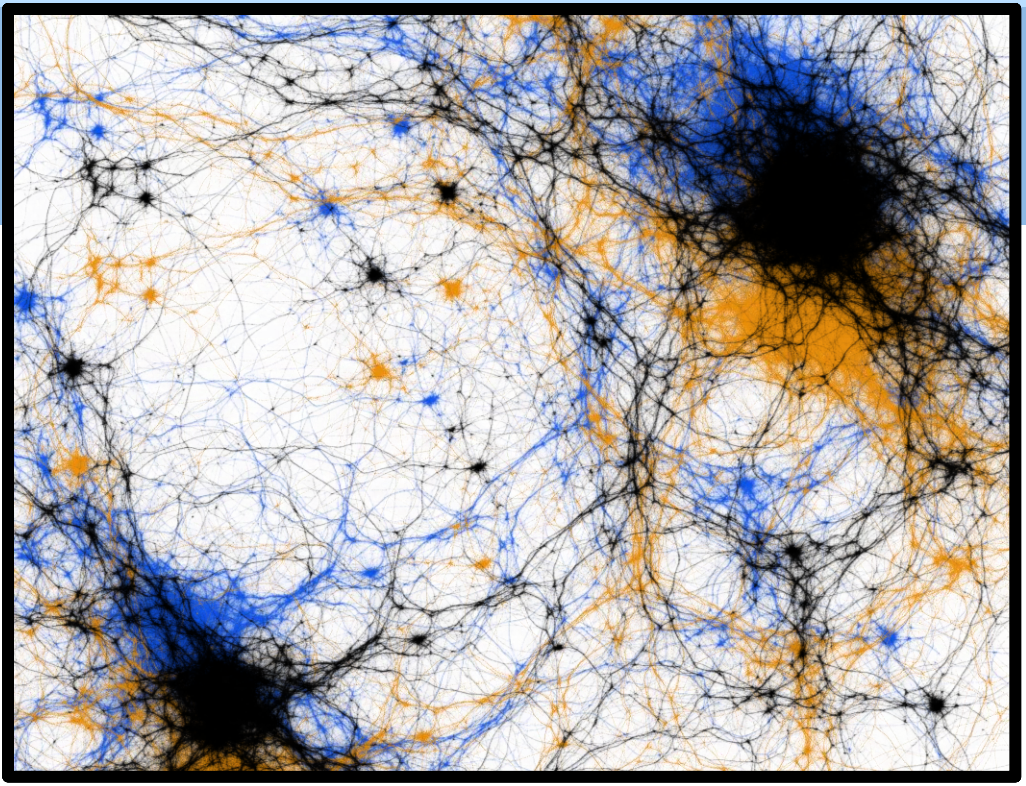
Matter overdensity tracers

Redshift

z

galaxy clustering

$$\delta^G(\vec{k}, z) = b^G(z)\, \delta_m(\vec{k}, z)$$
$$b^G(z) = a_1^G \sqrt{1+z}$$



Variable bias factors between matter overdensities and observables

Luminosity distance

D_L

dark sirens

$$\delta^W(\vec{k}, z) = b^W(z)\, \delta_m(\vec{k}, z)$$
$$b^W(z) = a_1^W (1+z)^{a_2^W}$$

C_ℓ-based likelihood

Angular power spectrum:

$$\delta^A(\vec{k}, z) = b^A(z)\, \delta_m(\vec{k}, z) \qquad W^{A_i}(z) = \frac{dn^{A_i}}{dz}\, H(z)\, b^A(z)$$

$$C^{A_iB_j}(\ell) = \int_{z_{\min}}^{z_{\max}} dz \frac{W^{A_i}(z)W^{B_j}(z)}{H(z)r^2(z)} \times P_{\delta\delta}\left[k = \frac{\ell + 1/2}{r(z)}, z\right] + N^{A_iB_j}(\ell)$$

Likelihood:

$$-2\ln\mathcal{L}(\vec{D}|\vec{\theta}) = \sum_{\ell=\ell_{\min}}^{\ell_{\max}} \left(\vec{D}_\ell - \vec{T}_\ell(\vec{\theta})\right)^T \mathcal{C}_\ell^{-1} \left(\vec{D}_\ell - \vec{T}_\ell(\vec{\theta})\right)$$

- $\vec{D}_\ell$: Data vector;
- $\vec{T}_\ell$: Theory vector for parameters $\vec{\theta}$;

$$\mathrm{Cov}\left[C^{A_iB_j}(\ell), C^{C_kD_n}(\ell')\right] = \frac{\delta_{\ell\ell'}}{(2\ell+1) f_{\rm fov} \Delta\ell} \left(C^{A_iC_k}(\ell)C^{B_jD_n}(\ell) + C^{A_iD_n}(\ell)C^{B_jC_k}(\ell)\right)$$

Cross-Correlation

$$-2\ln\mathcal{L}(\vec{D}|\vec{\theta}) = \sum_{\ell=\ell_{\min}}^{\ell_{\max}} \left(\vec{D}_\ell - \vec{T}_\ell(\vec{\theta})\right)^T \mathcal{C}_\ell^{-1} \left(\vec{D}_\ell - \vec{T}_\ell(\vec{\theta})\right)$$

$$\mathrm{Cov}\left[C^{AB}(\ell), C^{CD}(\ell)\right] = (\overrightarrow{AB}_\ell, \overrightarrow{CD}_\ell)$$

$$\mathcal{C}_\ell = \begin{pmatrix} (\overrightarrow{GG}_\ell, \overrightarrow{GG}_\ell) & (\overrightarrow{GG}_\ell, \overrightarrow{WG}_\ell) & (\overrightarrow{GG}_\ell, \overrightarrow{WW}_\ell) \\ (\overrightarrow{WG}_\ell, \overrightarrow{GG}_\ell) & (\overrightarrow{WG}_\ell, \overrightarrow{WG}_\ell) & (\overrightarrow{WG}_\ell, \overrightarrow{WW}_\ell) \\ (\overrightarrow{WW}_\ell, \overrightarrow{GG}_\ell) & (\overrightarrow{WW}_\ell, \overrightarrow{WG}_\ell) & (\overrightarrow{WW}_\ell, \overrightarrow{WW}_\ell) \end{pmatrix}$$

Cross-Correlation

$$-2\ln\mathcal{L}(\vec{D}|\vec{\theta}) = \sum_{\ell=\ell_{\min}}^{\ell_{\max}} \left(\vec{D}_\ell - \vec{T}_\ell(\vec{\theta})\right)^T \mathcal{C}_\ell^{-1} \left(\vec{D}_\ell - \vec{T}_\ell(\vec{\theta})\right)$$

$$\mathrm{Cov}\left[C^{AB}(\ell), C^{CD}(\ell)\right] = (\overrightarrow{AB}_\ell, \overrightarrow{CD}_\ell)$$

Galaxy clustering auto-correlation only (GG)

$$\mathcal{C}_\ell = \begin{pmatrix} (\overrightarrow{GG}_\ell, \overrightarrow{GG}_\ell) & (\overrightarrow{GG}_\ell, \overrightarrow{WG}_\ell) & (\overrightarrow{GG}_\ell, \overrightarrow{WW}_\ell) \\ (\overrightarrow{WG}_\ell, \overrightarrow{GG}_\ell) & (\overrightarrow{WG}_\ell, \overrightarrow{WG}_\ell) & (\overrightarrow{WG}_\ell, \overrightarrow{WW}_\ell) \\ (\overrightarrow{WW}_\ell, \overrightarrow{GG}_\ell) & (\overrightarrow{WW}_\ell, \overrightarrow{WG}_\ell) & (\overrightarrow{WW}_\ell, \overrightarrow{WW}_\ell) \end{pmatrix}$$

Cross-Correlation

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Gravitational Waves auto-correlation only (WW)

Cross-Correlation

$$-2\ln\mathcal{L}(\vec{D}|\vec{\theta}) = \sum_{\ell=\ell_{\min}}^{\ell_{\max}} \left(\vec{D}_\ell - \vec{T}_\ell(\vec{\theta})\right)^T \mathcal{C}_\ell^{-1} \left(\vec{D}_\ell - \vec{T}_\ell(\vec{\theta})\right)$$

$$\mathrm{Cov}\left[C^{AB}(\ell), C^{CD}(\ell)\right] = (\overrightarrow{AB}_\ell, \overrightarrow{CD}_\ell)$$

$$\mathcal{C}_\ell = \begin{pmatrix} (\overrightarrow{GG}_\ell, \overrightarrow{GG}_\ell) & (\overrightarrow{GG}_\ell, \overrightarrow{WG}_\ell) & (\overrightarrow{GG}_\ell, \overrightarrow{WW}_\ell) \\ (\overrightarrow{WG}_\ell, \overrightarrow{GG}_\ell) & (\overrightarrow{WG}_\ell, \overrightarrow{WG}_\ell) & (\overrightarrow{WG}_\ell, \overrightarrow{WW}_\ell) \\ (\overrightarrow{WW}_\ell, \overrightarrow{GG}_\ell) & (\overrightarrow{WW}_\ell, \overrightarrow{WG}_\ell) & (\overrightarrow{WW}_\ell, \overrightarrow{WW}_\ell) \end{pmatrix}$$

Cross-Correlation only (XC)

Cross-Correlation

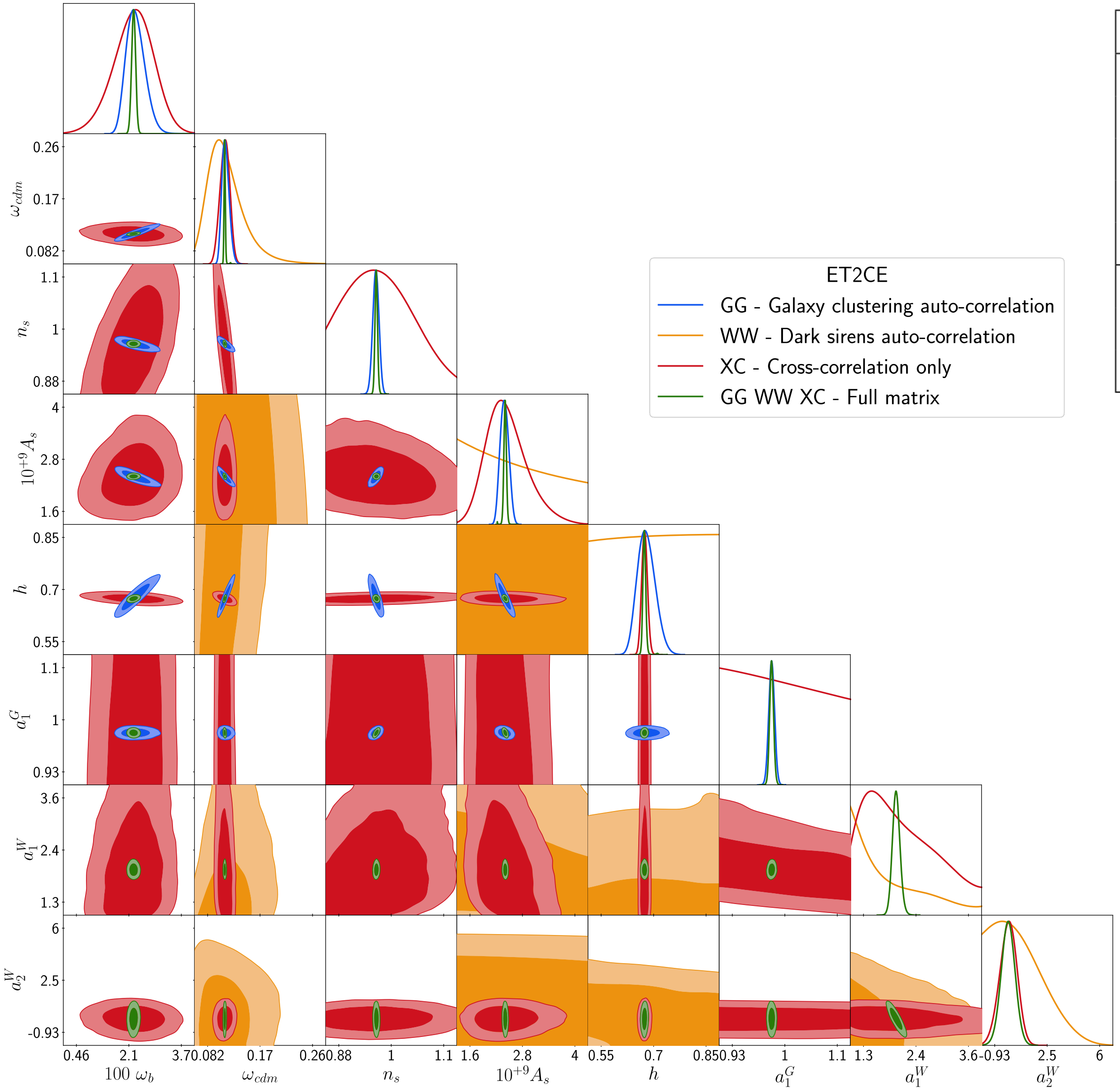
$$-2\ln\mathcal{L}(\vec{D}|\vec{\theta}) = \sum_{\ell=\ell_{\rm min}}^{\ell_{\rm max}} \left(\vec{D}_\ell - \vec{T}_\ell(\vec{\theta})\right)^T \mathcal{C}_\ell^{-1} \left(\vec{D}_\ell - \vec{T}_\ell(\vec{\theta})\right)$$

$$\mathrm{Cov}\left[C^{AB}(\ell), C^{CD}(\ell)\right] = (\overrightarrow{AB}_\ell, \overrightarrow{CD}_\ell)$$

$$\mathcal{C}_\ell = \begin{pmatrix} (\overrightarrow{GG}_\ell, \overrightarrow{GG}_\ell) & (\overrightarrow{GG}_\ell, \overrightarrow{WG}_\ell) & (\overrightarrow{GG}_\ell, \overrightarrow{WW}_\ell) \\ (\overrightarrow{WG}_\ell, \overrightarrow{GG}_\ell) & (\overrightarrow{WG}_\ell, \overrightarrow{WG}_\ell) & (\overrightarrow{WG}_\ell, \overrightarrow{WW}_\ell) \\ (\overrightarrow{WW}_\ell, \overrightarrow{GG}_\ell) & (\overrightarrow{WW}_\ell, \overrightarrow{WG}_\ell) & (\overrightarrow{WW}_\ell, \overrightarrow{WW}_\ell) \end{pmatrix}$$

Full matrix (GG WW XC)

Triangle plots

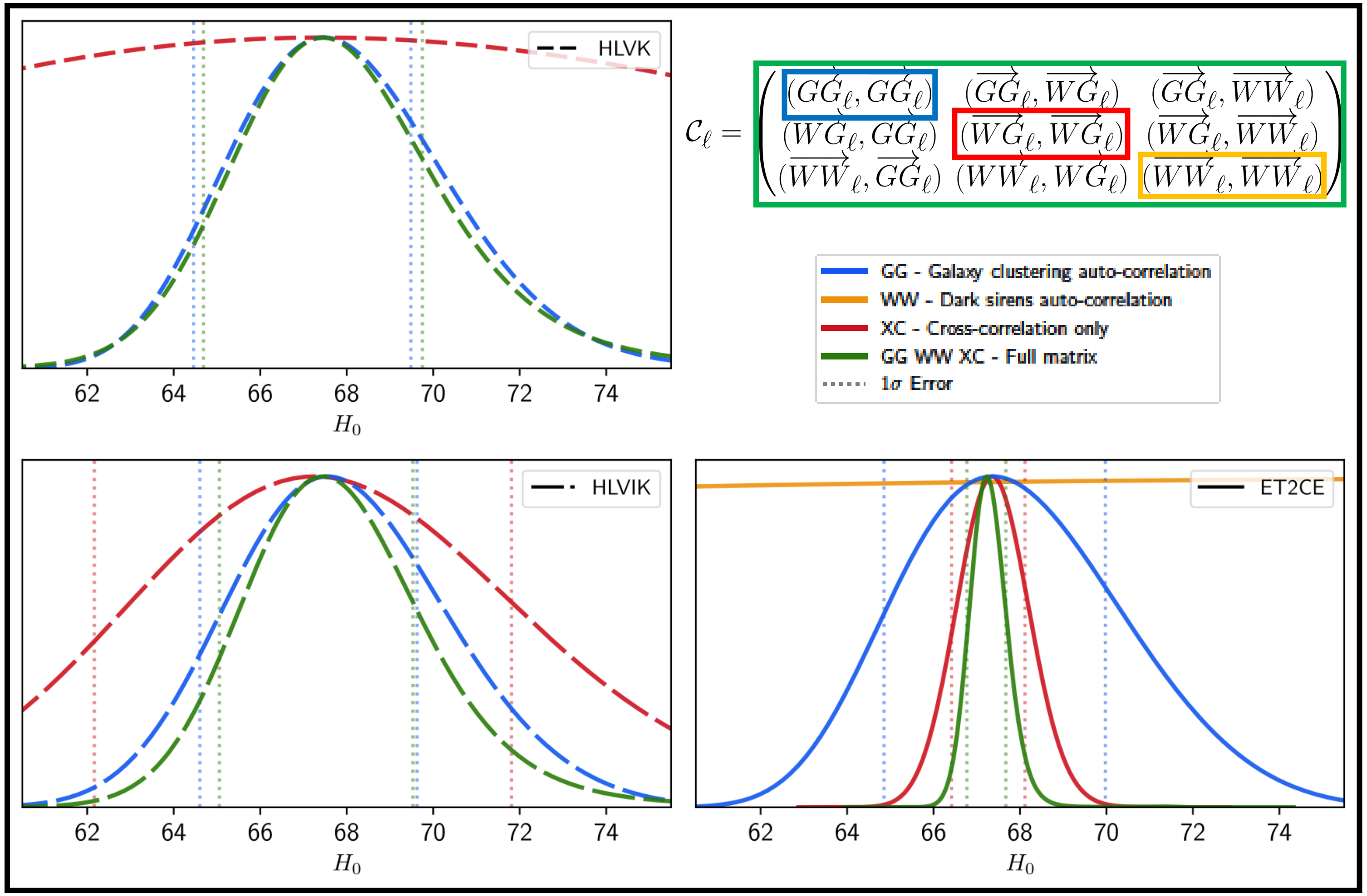


parameter	description	fiducial value	prior
ω_b	baryon density	2.249×10^{-2}	$[10^{-4}, 1]$
ω_cdm	cold dark matter density	0.112	$[10^{-2}, 0.5]$
n_s	scalar spectral index	0.96605	$[0.8, 1.2]$
A_s	scalar amplitude	2.42×10^{-9}	$[0.0, 10^{-7}]$
h	hubble parameter	0.6737	$[0.1, 1.5]$
a_1^G	galaxy clustering bias normalization	1.0	$[0.5, 2.0]$
a_1^W	GW bias normalization	2.0	$[0.0, 4.0]$
a_2^W	GW bias slope	0.0	$[-2.0, 7.0]$

<u>Posteriors on cosmological parameters by running MCMC chains:</u>

- H_0 is the only parameter that takes advantage from the cross-correlation;
- Dark sirens auto-correlation has no constraining power.

Results on H_0



- HLVK and HLVIK barely improve the constraint given by galaxy clustering only, due to the small GW catalogue;

- The larger ET2CE GW catalogue ($\sim 200'000$ events) leads to a precise estimation of the Hubble parameter:

$$H_0 = 67.2^{+0.5}_{-0.4}$$

- Cross-correlation drives the constraint for ET2CE.

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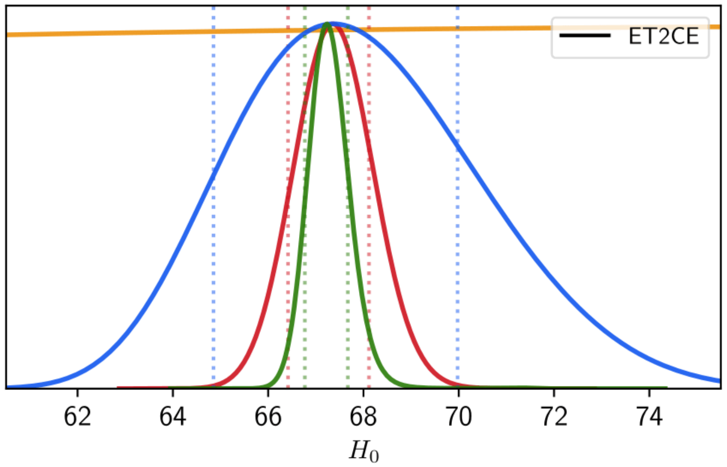
Conclusions

- Cross-correlation is an assumption-light technique that can be used to infer the Hubble parameter with precision;

- Decomposable full likelihood analysis;

- It requires a large catalogue to statistically infer parameters efficiently. Only ET2CE will detect a sufficiently large sample of dark sirens to improve H_0 constraint, better than 1% at 1σ.

$$H_0 = 67.2^{+0.5}_{-0.4}$$



Future studies

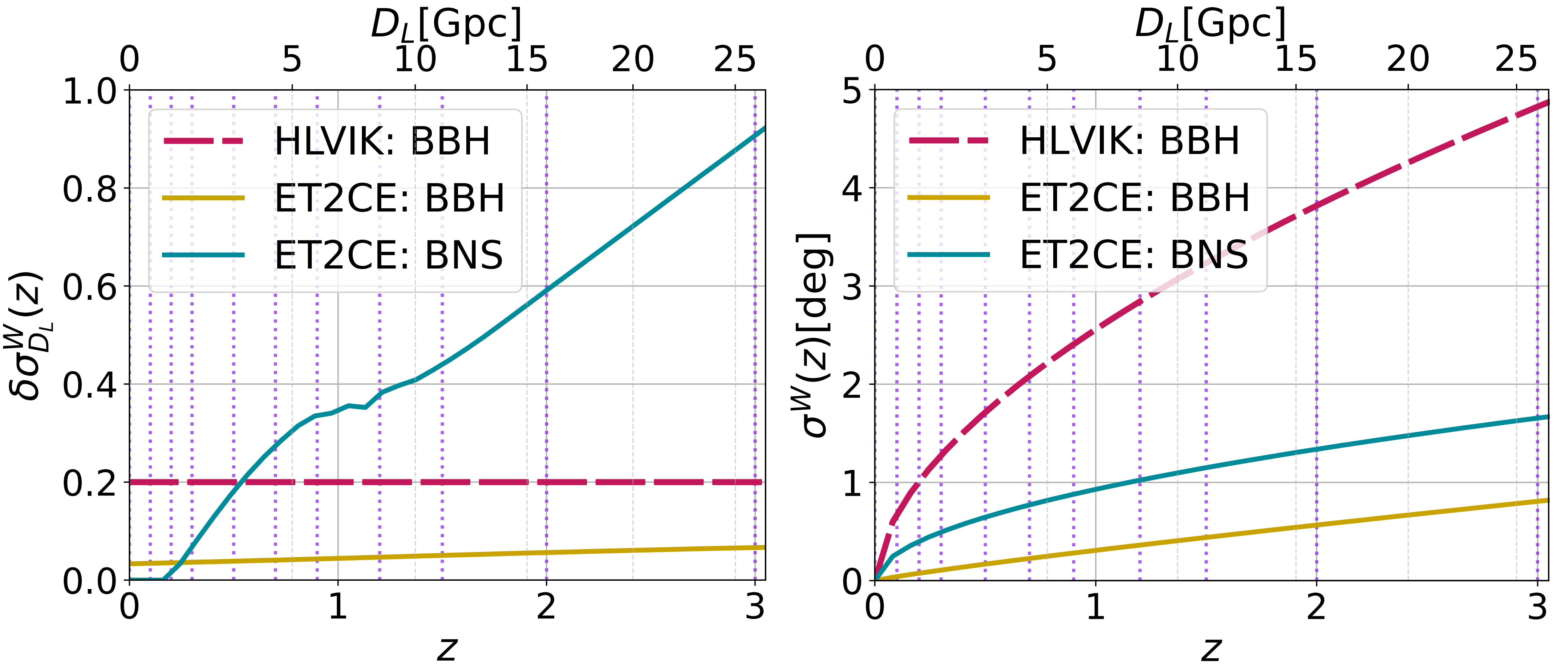
- Beyond LVK frequencies, e.g. LISA;
- New techniques, or combinations of techniques, e.g. peak sirens (Ferri et al. 2412.00202, Matos et al. 2512.15380);
- Correlating different or more observables (e.g. De Leo et al. 2512.19186);
- Beyond H_0, cross-correlation is useful for inferring other cosmological parameters.

Thank you for your attention

Questions?

Backup slides

GW uncertainties



Angular power spectrum

Angular power spectrum:

$$C^{A_iB_j}(\ell) = \int_{z_{\min}}^{z_{\max}} dz \frac{W^{A_i}(z)W^{B_j}(z)}{H(z)r^2(z)} \times P_{\delta\delta}\left[k = \frac{\ell + 1/2}{r(z)}, z\right] + N^{A_iB_j}(\ell)$$

- Computed with Limber approximation;
- Window function:

$$W^{A_i}(z) = \frac{dn^{A_i}}{dz} H(z)\ b^A(z)$$

- Noise in the auto-correlation, including shot noise and angular resolution.

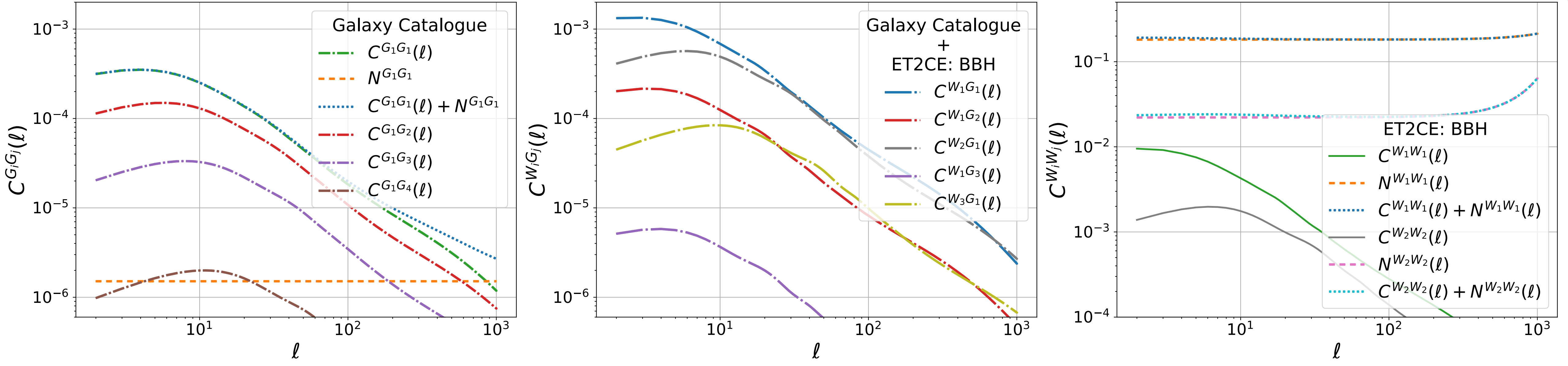
$$N^{A_iB_j}(\ell) = \frac{4\pi f_{\rm sky}^A}{N^{A_i}} \frac{\delta_{A_iB_j}}{\left(\mathcal{W}^{A_i}(\ell)\right)^2} \qquad \mathcal{W}^{A_i}(\ell) = \exp\left(-\frac{(\sigma^{A_i})^2\,\ell^2}{2}\right)$$

C_ℓs

Angular power spectrum:

$$\delta^A(\vec{k}, z) = b^A(z)\,\delta_m(\vec{k}, z) \qquad W^{A_i}(z) = \frac{dn^{A_i}}{dz}\,H(z)\,b^A(z)$$

$$C^{A_iB_j}(\ell) = \int_{z_{\min}}^{z_{\max}} dz \frac{W^{A_i}(z)W^{B_j}(z)}{H(z)r^2(z)} \times P_{\delta\delta}\left[k = \frac{\ell+1/2}{r(z)}, z\right] + N^{A_iB_j}(\ell)$$



C_ℓ-based likelihood

Likelihood:

$$-2\ln\mathcal{L}(\vec{D}|\vec{\theta}) = \sum_{\ell=\ell_{\min}}^{\ell_{\max}} \left(\vec{D}_\ell - \vec{T}_\ell(\vec{\theta})\right)^T \mathcal{C}_\ell^{-1} \left(\vec{D}_\ell - \vec{T}_\ell(\vec{\theta})\right)$$

- $\vec{D}_\ell$: Data vector;
- $\vec{T}_\ell$: Theory vector for parameters $\vec{\theta}$;

$$\vec{V}_\ell = \begin{pmatrix} \overrightarrow{GG}_\ell \\ \overrightarrow{WG}_\ell \\ \overrightarrow{WW}_\ell \end{pmatrix}$$

- Assuming a Gaussian distribution of the spherical harmonic coefficients:

$$\mathrm{Cov}\left[\vec{T}_\ell(\vec{\theta}), \vec{T}_{\ell'}(\vec{\theta})\right] = \mathrm{Cov}\left[C^{A_iB_j}(\ell), C^{C_kD_n}(\ell')\right] = \frac{\delta_{\ell\ell'}}{(2\ell+1) f_{\mathrm{fov}} \Delta\ell} \left(C^{A_iC_k}(\ell) C^{B_jD_n}(\ell) + C^{A_iD_n}(\ell) C^{B_jC_k}(\ell)\right)$$

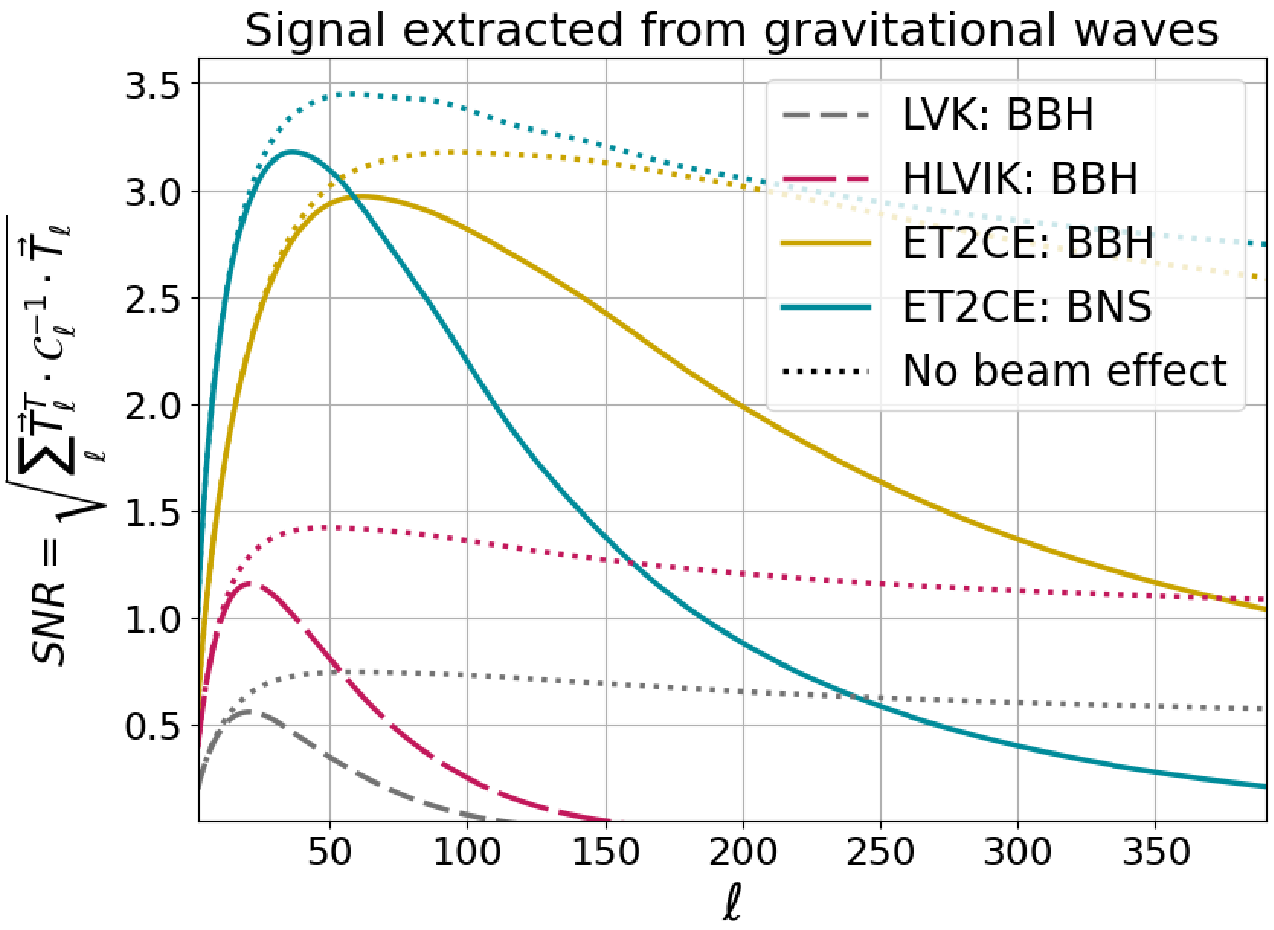
$$\mathrm{Cov}\left[C^{AB}(\ell), C^{CD}(\ell)\right] = (\overrightarrow{AB}_\ell, \overrightarrow{CD}_\ell)$$

Signal-to-noise ratio

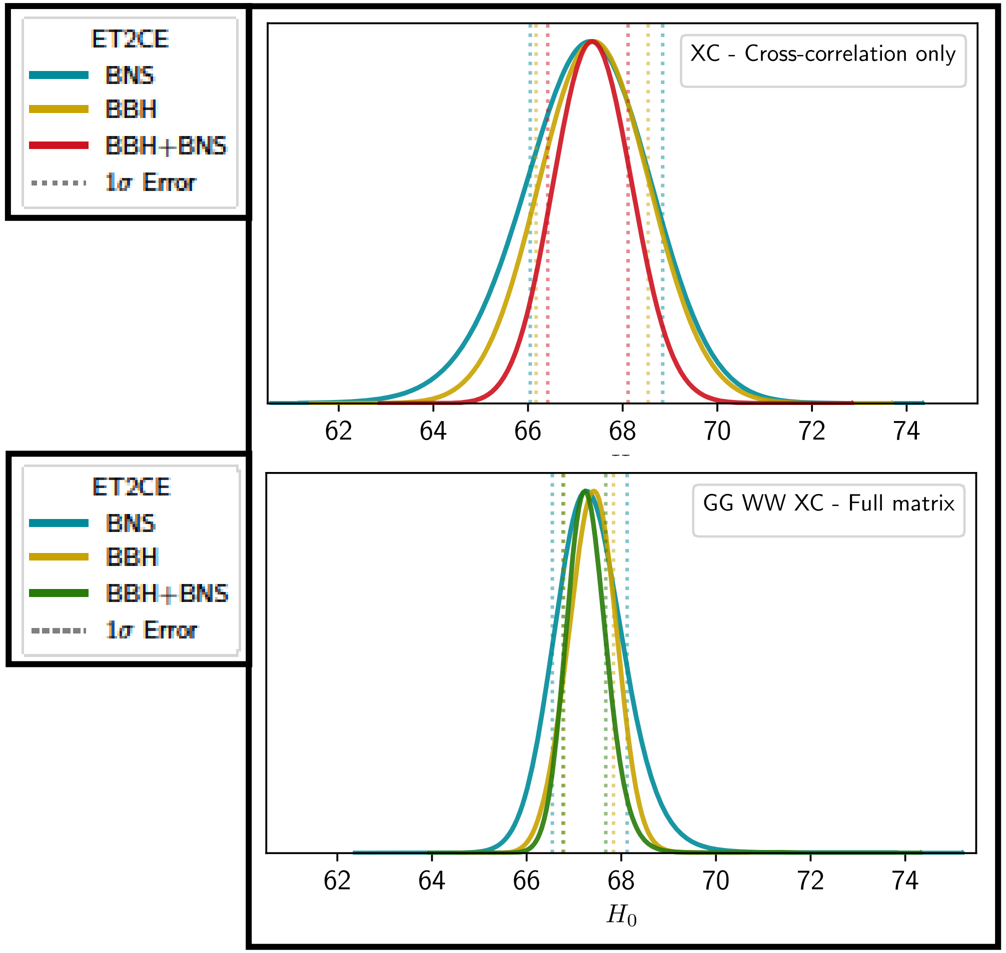
- $\vec{D}_\ell = 0$: Data vector;
- $\vec{T}_\ell$: Theory vector for parameters $\vec{\theta}$;

- Total signal extracted from the gravitational waves detections correlation with galaxy clustering.

$$(\mathrm{SNR})^2 = \sum_{\ell=\ell_{\min}}^{\ell_{\max}} \vec{T}_\ell(\vec{\theta}_{\mathrm{fid}})^T \, \mathcal{C}_\ell^{-1} \, \vec{T}_\ell(\vec{\theta}_{\mathrm{fid}})$$



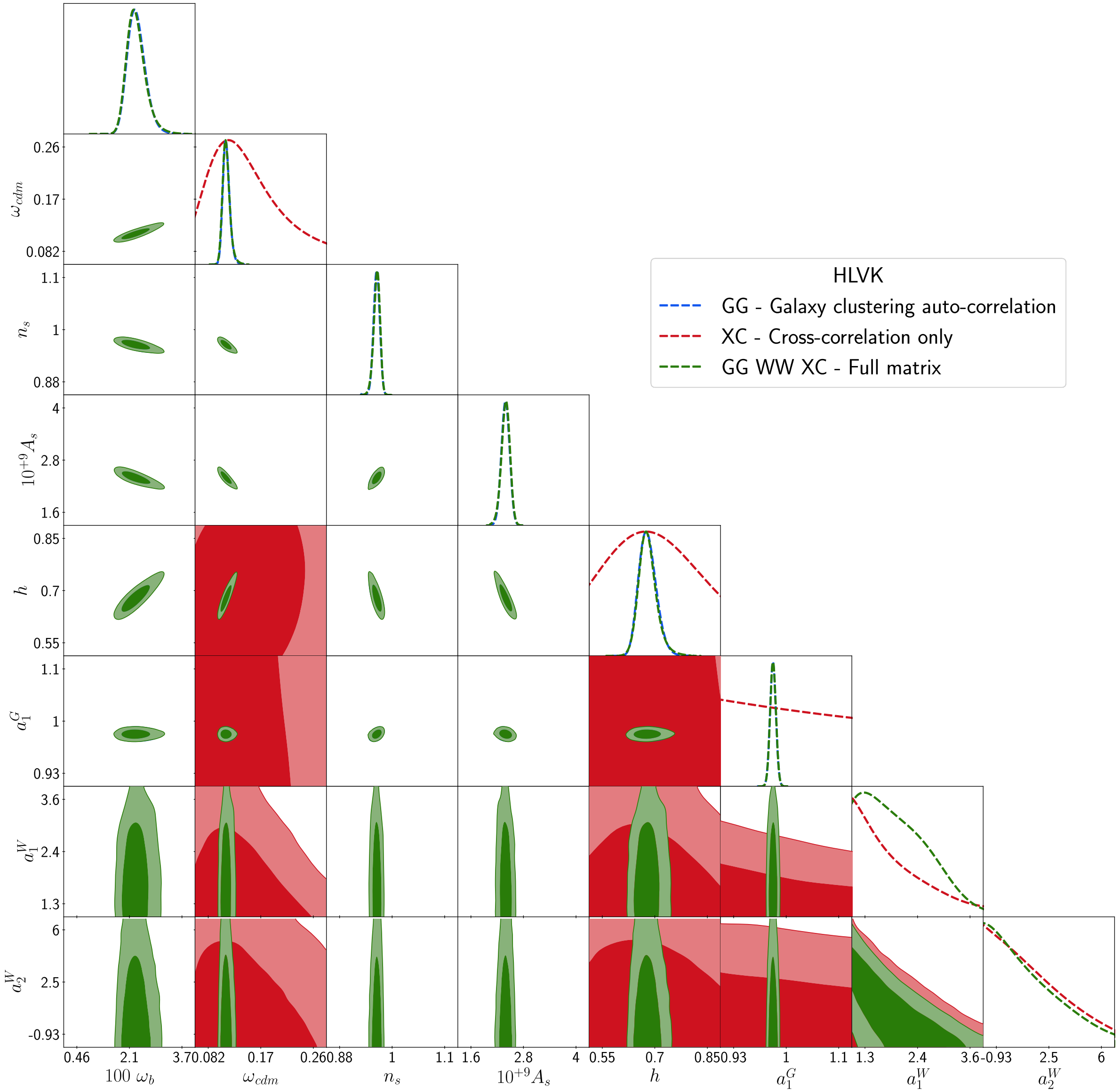
BBH-BNS comparison



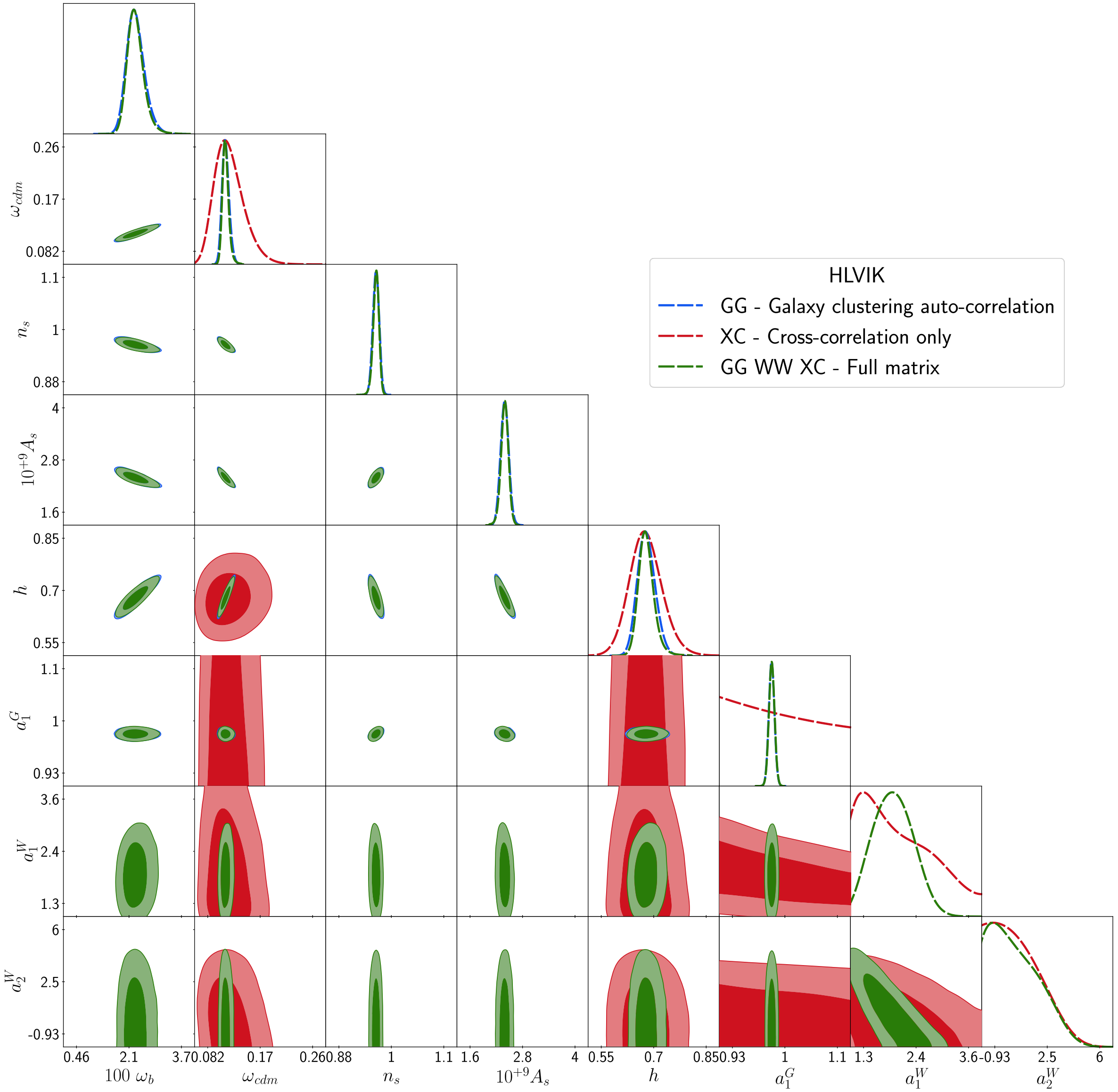
Both BBH and BNS are considered for ET2CE:

- BNS sample is optimistic and larger than BBH;
- BBH sample has slightly more constraining power leading the combined results.

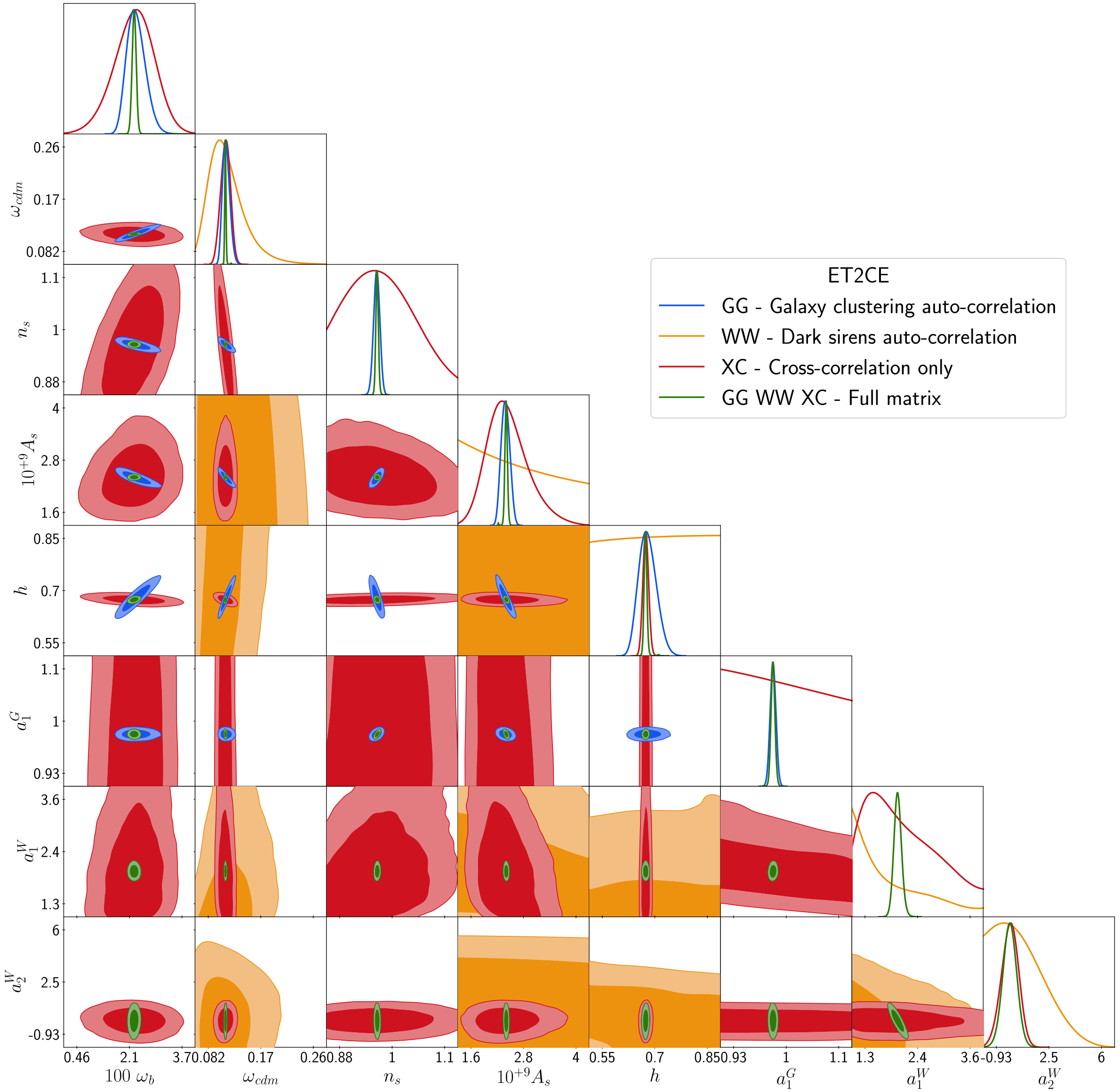
Triangle plots



Triangle plots



Triangle plots



a_{lm}-C_ℓ comparison